DEPARTMENT OF MATHEMATICS

Scheme & Syllabi

Ph.D. course work in Mathematics

W.E.F. 2019-20

CENTRAL UNIVERSITY OF HARYANA
JANT-PALI, MAHENDERGARH
HARYANA-123031
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Department of Mathematics

Central University of Haryana
Mahendergarh, Haryana-123031

Scheme and Syllabus of Ph.D. Mathematics
(CHOICE BASED CREDIT SYSTEM)

Course Type

Core Course (C)
Discipline Centric Elective Course (E) of subject specific.

Total Credit: 12

Each candidate is required to take core course and one elective course of subject specific.

S. No.	Course Title	Course Code	Credits	Course Type
1.	SPMS MAT 02 01 01 C 6006	Research Methodology	6	Core
2.	SPMS MAT 02 01 01 E 5106	Symmetries and Differential Equations	6	Elective
3.	SPMS MAT 02 01 02 E 5106	Lie Groups and Lie Algebra	6	Elective
4.	SPMS MAT 02 01 03 E 5106	Fractional Calculus and Fractional Differential Equations	6	Elective
5.	SPMS MAT 02 01 04 E 5106	Finite Element Analysis	6	Elective
6.	SPMS MAT 02 01 05 E 5106	Mathematical Foundation of Image Processing	6	Elective
7.	SPMS MAT 02 01 06 E 5106	Approximation Theory	6	Elective
8.	SPMS MAT 02 01 07 E 5106	Complex Dynamics	6	Elective
9.	SPMS MAT 02 01 08 E 5106	Dynamical Systems	6	Elective
10.	SPMS MAT 02 01 09 E 5106	Computational Neuroscience	6	Elective
11.	SPMS MAT 02 01 10 E 5106	Algebraic Topology	6	Elective

S. No.	Course Title	Course Code	Credits	Course Type
12.	SPMS MAT 02 01 11 E 5106	Knot Theory	6	Elective

Research Methodology

(SPMS MAT 02 01 01 C 6006)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Research Problems: Meaning, Motivation, Objectives and types of research, Significance of research, Research proposals and aspects, Criteria of good research, Research formulation and hypotheses, Selection and necessity of defining the problem, Literature review, Primary and secondary sources, Reviews, Treatise, Monographs, Patents.

UNIT II

Research Design: Need, Problem Definition, Variables, Research design concepts, Research design process, Research Modeling: Types of models, Model building and stages, Data collection, processing and analysis, Simulation techniques using computer software(s).

UNIT III

Design and Planning of Experiments: Aims and objectives, expected outcome, methodology to be adopted, importance of reproducibility of research work, Interpolation, Extrapolation, Types of errors (rounding, truncation, machine and random), Error analysis and least square curve fitting. Analysis of Variance components (ANOVA) for fixed effect model, Objectives and basic principles of designs of experiments. Complete randomized design (CRD), Randomized block design (RBD) and Latin square design (LSD).

UNIT IV

Data mining and Report Writing: Library resources, Internet, Scientific search engines, Introduction to Latex/Google docs, Structure and component of research paper, Presenting the research paper/thesis, Journal impact factor, Citation index, References and bibliography, Copyright, Plagiarism and ethics in research, Communication and presentation.

Suggested Readings:

1. Kothari, C.R & Garg, G. (2014). Research Methodology: Methods and Techniques, 3rd Edition. New Age International Publishers, New Delhi.
2. Pannerselvan, R. (2009). Research Methodology. Prentice Hall of India, New Delhi.
3. Singh, Y.K. (2008). Fundamental of Research Methodology and Statistics. New Age International Publishers, New Delhi.
4. Montgomery, D.C. (2013). Design and Analysis of Experiments, 8th Edition. Wiley India.
5. Prathapan, K. (2014). Research Methodology for Scientific Research. IK International, New Delhi.

Symmetries and Differential Equations

(SPMS MAT 0**2** 01 01 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Dimensional Analysis: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables.

UNIT II

Lie Group of Transformations: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators.

UNIT III

Invariant functions, Canonical coordinates, Lie bracket, Lie series, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations- one dependent and n independent variables.

UNIT IV

Group theoretic techniques, Lie Classical method, Symmetry reduction method, Non classical method, Reduction of PDEs to Ordinary differential equations (ODEs), Exact solutions of PDEs.

Suggested Readings:

1. Bluman G. W. and Anco A. C., *Symmetry and Integration Methods for Differential Equations*, Appl. Math. Sci., 154, Springer, New York, 2002.
2. Bluman G. W. and Kumei S., *Symmetries and Differential Equations*, Appl. Math.Sci., Springer-Berlin, 1989.
3. Olver P. J., *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York 1993. 4. L. V. Ovsiannikov, *Group Properties of Differential Equations*, Novosibirsk, Moscow, 1962.

Lie Groups and Lie Algebra

(SPMS MAT 0**2** 01 02 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Differential Manifolds, Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

UNIT II

Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Multi-parameter Lie groups of transformations; Lie algebra and the exponential map, Adjoint representation.

UNIT III

Lie algebras, r-parameter Lie groups of transformations, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems.

UNIT IV

Semi-simple and reductive algebras, Semi-simplicity of classical Lie algebras, Killing form; Jordan decomposition; Engel's Theorem, Cartan subalgebra and Root space decomposition.

Suggested Readings:

1. J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, Lie Algebras, Wiley-Interscience, New York, 1962.
3. J. P. Serre, Lie Algebras and Lie Groups, Benjamin, New York, 1965.
4. N. Bourbaki, Lie Groups and Lie Algebras, Springer Science & Business Media, 1998.
5. K. J. Alexander, An Introduction to Lie Groups and Lie Algebras, Cambridge University Press. 6. S. Kumaresan, Differential Geometry and Lie Groups, Hindustan Book Agency.
7. B. Hall, Lie Groups, Lie Lagebras, and Representations: An Elementary Introduction, Second Edition, Springer.
8. P. J. Olver, Application of Lie Groups to Differential Equations, Second Edition, Springer.

Fractional Calculus and Fractional Differential Equations

(SPMS MAT 0**2** 01 03 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Gamma function and its properties, Beta function, Contour integral representation. Fractional derivatives: Grunwald-Letnikov, Riemann-Liouville and Caputo’s fractional derivative, Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

UNIT II

Sequential fractional derivatives. Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms and Mellin transforms of fractional derivatives.

UNIT III

Linear Fractional Differential Equations: Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard fractional differential equations. Sequential fractional differential equations.

UNIT IV

Fractional Differential Equations: Introduction, Linearly independent solutions, Solutions of the homogeneous and non-homogeneous fractional differential equations, Reduction of fractional partial differential equations to ordinary differential equations.

Suggested Readings:

1. Oldham K. B. & Spanier J., The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order, Dover Publications Inc, 2006.
2. Miller K. S. & Ross. B., An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover, Wiley Blackwell, 1993.
3. Podlubny I., Fractional Differential Equations, Academic Press, 1998.

Finite Element Analysis

(SPMS MAT 0**2** 01 04 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Introduction to Finite Element Methods. Difference between Finite Element and Finite Difference Methods, Method of Weighted Residuals: Collocation, Method of Least squares, Galerkin's method, Review of Calculus of Variations, Ritz Method.

UNIT II

Linear, quadratic and higher order elements, Difference between 1D and 2D approach, types of 2D elements, Local coordinates, Global coordinates, Assembly. Triangular elements: linear and quadratic elements with area coordinates, General Quadrilateral elements, linear and higher order.

UNIT III

Numerical integration, Construction of shape functions: Linear elements (One dimensional bar element, two dimensional-Triangular and rectangular elements, three dimensional tetrahedron element). Assembly of element equations and their solution, Application to flow and heat transfer problems, Discussion over higher order differential equations.

UNIT IV

Application of finite element methods for solving various boundary value problems, Computer procedures for Finite element analysis.

Suggested Readings:

1 . Zienkiewicz, O. C. and Taylor, R. L. *The Finite Element Method: The Basis.* Butterworth-Heinemann, 2000.

2. Smith, G. D. *Numerical solution of Partial Differential Equations: Finite difference methods*. Oxford Applied Mathematics and Computing Science Series, 1985.

3. Hughes, T. J. R. *The Finite Element Method (Linear Static and Dynamic Finite Element Analysis).* Courier Corporation, 2007.

Mathematical Foundation of Image Processing

(SPMS MAT 0**2** 01 05 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Image: The origins of digital image processing, classification of images, examples of fields that use digital image processing, elements of digital image processing systems, image sampling and quantization, some basic relationships like neighbours, connectivity, distance measures between pixels translation, scaling, rotation and perspective projection of image, reading, displaying, writing images using software. data classes, image types using software based simulation.

UNIT II

Operations on Images: Converting between data classes and image types, some basic gray level transformations histogram processing, enhancement using arithmetic and logic operations combining spatial enhancement methods basics of spatial filters, smoothening and sharpening spatial filters intensity transformation function, histogram processing and function plotting using software.

UNIT III

Transforms on Images: Introduction to Fourier transform and the frequency domain. Computing and visualizing image in frequency domain, Smoothing Frequency Domain Filters, Sharpening Frequency Domain Filters, Homomorphic Filtering. Mathematical decomposition on images. Wavelet transform on images.

UNIT IV

Image restoration: A model of the image degradation / restoration process, noise models restoration in the presence of noise only spatial filtering, periodic noise reduction by frequency domain filtering linear position-invariant degradations estimation of degradation function, inverse filtering wiener filtering, geometric mean filter geometric transformations, image compression: coding interpixel and psychovisual redundancy image compression models compression standards, image segmentation: detection of discontinuities edge linking and boundary detection thresholding, object recognition: patterns and pattern classes decision-theoretic methods structural methods.

Suggested Readings:

1. Jain A. K., *Fundamentals of Digital Image Processing*, 1 edition. Englewood Cliffs, NJ: Pearson, 1988.
2. Lim J. S., *Two-Dimensional Signal and Image Processing*, 1 edition. Englewood Cliffs, N.J: Prentice Hall PTR, 1989.
3. Gonzalez R. C., Woods R. E., and Eddins S. L., *Digital Image Processing Using Matlab*, 1st edition. New Delhi: Dorling Kindersley Pvt Ltd, 2006.
4. Gonzalez, *Digital Image Processing*, 4th ed. Upper Saddle River, N.J.: Pearson, 2018.
5. Gong S., Liu C., Ji Y., Zhong B., Li Y., and Dong H., *Advanced Image and Video Processing Using MATLAB*. Springer International Publishing, 2019.

Approximation Theory

(SPMS MAT 02 01 06 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT-I

Concept of best approximation in a normed linear space, Existence of the best approximation, Uniqueness problem, Convexity-uniform convexity, strict convexity and their relations, Continuity of the best approximation operator.

UNIT-II

The Weierstrass theorem, Bernstein polynomials, Korovkin theorem, Algebraic and trigonometric polynomials of the best approximation, Lipschitz class, Modulus of continuity, Integral modulus of continuity and their properties.

UNIT-III

Bernstein's inequality, Jackson's theorems and their converse theorems, Approximation by means of Fourier series.

UNIT-IV

Positive linear operators, Monotone operators, Simultaneous approximation, L^p -approximation, Approximation of analytic functions.

Suggested Books:

1. Cheney, E. W., "Introduction to Approximation Theory", AMS Chelsea Publishing Co. 1981.
2. Lorentz, G. G., "Bernstein Polynomials", Chelsea Publishing Co. 1986
3. Natanson, I. P., "Constructive Function Theory Volume-I", Fredrick Ungar Publishing Co 1964.
4. Mhaskar, H. M. and Pai, D. V., "Fundamentals of Approximation Theory" Narosa Publishing House 2000
5. Timan, A. F., "Theory of Approximation of Functions of a Real Variable," Dover Publication Inc 1994.
6. Gupta V. and Agarwal, R. P., Convergence Estimates in Approximation Theory, Springer, 2014.

Complex Dynamics

(SPMS MAT 02 01 07 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT-I

Iteration of a Mobius transformation, attracting, repelling and indifferent fixed points. Iterations of R(z) = z^2 , z^2+c, z + 1/z. The extended complex plane, chordal metric, spherical metric, rational maps, Lipschitz condition, fixed points, Critical points, Riemann Hurwitz relation.

UNIT-II

Equicontinuous functions, normal sets, Fatou sets and Julia sets, completely invariant sets, Normal families and equicontinuity, Properties of Julia sets, exceptional points backward orbit, minimal property of Julia sets.

UNIT-III

Julia sets of commuting rational functions, structure of Fatou set, Completely invariant components of the Fatou set, The Euler characteristic, Riemann Hurwitz formula for covering maps, maps between components of the Fatou sets, the number of components of Fatou sets, components of Julia sets.

UNIT-IV

Origin of bifurcation, stability of fixed point, equilibrium point, Hopf bifurcation, Non linear oscillator, solution of non-linear differential equations, randomness of orbits of a dynamical system, chaos, strange attractors. Classical Fractals, Cantor set, Sierpinski triangle, Von Koch curve.

Suggested references:

1. An introduction to chaotic dynamical system by Robert L. Devaney, Addison Wesley publishing house Co inc. 1989.
2. Fractals Everywhere by M. F. Barnsley, 2nd edition, Academic Press, 1995
3. Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering, by Steven H. Strogatz, Westview Press.
4. A. F. Beardon, Iteration of rational functions, Springer Verlag , New York, 1991.
5. S. Morosawa, Y. Nishimura, M. Taniguchi, T. Ueda, Holomorphic dynamics, Cambridge University Press, 2000.

Dynamical System

(SPMS MAT 0**2** 01 08 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT-I

Linear Systems: Exponentials of operators, linear systems in **R2,** Complex eigenvalues, Multiple eigenvalues, Jordon forms, Stability theory, generalized eigenvectors and invariant subspaces, Non-homogeneous linear systems.

UNIT-II

Non-linear Systems: local analysis: the fundamental existence-uniqueness theorem, the flow defined by a differential equation, Linearization, Stability and Liapunov functions, Saddles, Nodes, Foci, and Centers. Global analysis: Dynamical systems and global existence theorem, Limit sets and Attractors, Periodic orbits, Limit Cycles, and Seperatrix cycles, the Poincare map, Poincare-Bendixon theory in R2.

UNIT-III

Origin of bifurcation, stability of fixed point, equilibrium point, Hopf bifurcation, Non linear oscillator, solution of non-linear differential equations, randomness of orbits of a dynamical system, chaos, strange attractors.

UNIT-IV

Iteration theory, Julia sets, Fatou sets, Mandelbrot set, Characterization of Julia sets, Dynamics of functions exp z , sin z and cos z, Classical Fractals, Cantor set, Sierpinski triangle, Von Koch curve.

Suggested references:

1. Differential Equations and Dynamical Systems by Lawrence Perko, Springer-Verlag, 2006.
2. Differential Equations, Dynamical Systems and an Introduction to Chaos by Morris W. Hirsch, Stephen Smale and Robert L. Devaney, Academic Press, 2013
3. Dynamical Systems and Numerical Analysis by A.M. Stuart and A.R. Humphries, Cambridge University Press, 1998.
4. Fractals Everywhere by M. F. Barnsley, 2nd edition, Academic Press, 1995
5. An introduction to chaotic dynamical system by Robert L. Devaney, Addison Wesley publishing house Co inc. 1989.
6. Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry and Engineering, by Steven H. Strogatz, Westview Press.
7. A. F. Beardon, Iteration of rational functions, Springer Verlag , New York, 1991.
8. S. Morosawa, Y. Nishimura, M. Taniguchi, T. Ueda, Holomorphic dynamics, Cambridge University Press, 2000.

Computational Neuroscience

(SPMS MAT 0**2** 01 09 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT I

Dynamical System: Preliminaries from linear algebra, Bases and subspaces, Planer linear systems and its classification, eigenvalues and eigenvector, solving linear systems, phase portraits for planer system, trace-determinant plane, dynamical classification.

UNIT II

Nonlinear systems: Dynamic systems, the existence and uniqueness theorem, continuous dependence of solutions, the variational equation, Equilibria in non-linear systems and some illustrative examples, nonlinear sinks and sources, saddles, stability, bifurcation, nullclines, Software based simulation.

UNIT III

Neuron: Elements of Neuronal systems, the ideal spiking neuron, spike trains, synapses, postsynaptic potentials, firing threshold and action potential, a phenomenological neuron model, The problem of neuronal coding, rate code as spike count, spike density, population activity, spike code as time-to-first-spike, phase, correlations and synchrony, stimulation reconstruction and revers correlation. Nernst potential, reversal Potential.

UNIT IV

Single Neuron Modeling: Hodgkin-Huxley model, definition and dynamics and its software based simulation. Effect of various Ion channels, Sodium channel, potassium channel, low-threshold Calcium channel and its dynamics. Feedback: inhibitory and excitatory synapses. Two-dimensional Neuron models and their software based simulation.

Suggested references:

1. Gerstner W. and Kistler W. M., *Spiking Neuron Models: Single Neurons, Populations, Plasticity*, 1 edition. Cambridge University Press, 2002.
2. 2. Feng J., *Computational Neuroscience: A Comprehensive Approach*. CRC Press, 2003.
3. Dayan P., Sejnowski T. J., and Poggio T. A., *Theoretical Neuroscience – Computational and Mathematical Modeling of Neural Systems*, Revised ed. edition. Cambridge, Mass.: MIT Press, 2005.
4. Perko L., *Differential Equations and Dynamical Systems*, 3rd edition. New York: Springer, 2006.
5. Lynch S., *Dynamical Systems with Applications using Mathematica®*, 2007 edition. Boston, Mass: Birkhäuser, 2007.
6. Trappenberg T., *Fundamentals of Computational Neuroscience*, 2 edition. OUP Oxford, 2009.
7. Hirsch M. W., Smale S., and Devaney R. L., *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, 3 edition. Waltham, MA: Academic Press, 2012.
8. Robertson K., Ed., *Computational Neuroscience*. Larsen and Keller Education, 2017.
9. Miller P., Sejnowski T. J., and Poggio T. A., *An Introductory Course in Computational Neuroscience*, 1 edition. Cambridge, Masachusetts: MIT Press, 2018.

Algebraic Topology

(SPMS MAT 0**2** 01 10 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT-I

Mainfolds: Indentification (quotient) spaces and identification (quotient) maps; topology n-manifolds including surfaces, Sn, RPn, CPn. and lens spaces. Triangulated manifolds: Representation of triangulated, closed 2-manifolds as connected sums of tori of projective planes.

UNIT-II

Fundamental group, functoriality, retract, deformation retract; Van Kampen's Theorem, classification of surfaces by abelianizing the fundamental group, covering spaces, path lifting, homotopy lifting, uniqueness of lifts, general lifting theorem for maps.

UNIT-III

Covering transformations, regular covers, correspondence between subgroups of the fundamental group and covering spaces, computing the fundamental group of the circle, RPn, lens spaces via covering spaces.

UNIT-IV

Simplicial homology: Homology groups, functoriality, topological invariance, Mayer-Vietoris sequence; applications, including Euler characteristic, classification of closed trangulated surfaces via gomology and via Euler characteristic and orientability; degree of a map between oriented manifolds, Lefschetz number, Brouwer Fixed Point Theorem.

Suggested Readings:

1. A Hatcher. Algebraic Topology, Cambridge University Press, 2002.
2. R C Kirby and L C Siebenmann. Foundational Essays on Topological Manifolds, Smoothings, and Triangulations. Annals of Math Studies 88. Princeton University Press, 1977.
3. J P May. A Concise Course in Algebraic Topology. University of Chicago Press, 1999.
4. G E Bredon. Topology and Geometry. Springer GTM 139, 1993.

Knot Theory

(SPMS MAT 0**2** 01 11 E 5106)

End Semester Examination: 60
Internal Assessment: 40
Total: 100

UNIT-I

Knots, diagram of a knot, Composition of knots, Reidemeister moves, links, Invariants of knots, rational tangles, 2-bridge knots.

UNIT - II

Surfaces and Knots: Genus and Seifert surfaces, Torus knots, knots and its properties, Setelite Knots, Hyperbolic Knots.

UNIT – III

Braids, Braids groups, Morkov moves, Alexander's theorem, Representation of torus knots, Alternating knots, Quasi alternating knots.

UNIT-IV

Alexander polynomial, Bracket polynomial, HOMFLY polynomial, Jones polynomial, Vassiliev Invariants, Knot complements and 3-Manifolds.

Suggested Readings:

1. C C Adams. The Knot Book. W.H. Freeman, 1994
2. C Livingston. Knot Theory. Carus Mathematical Monographs 24. Mathematical Association of America, 1993.
3. J Roberts. Knots Knotes. http://math.ucsd.edu/˜justin/papers/knotes.pdf
4. D Rolfsen. Knots and Links. Publish or Perish, 1976.
5. G Burde and H Zieschang. Knots. 2nd ed. de Gruyter, 2003.
6. A Kawauchi. Survey of Knot Theory. Birkhauser, 1996.
7. W B R Lickorish. An Introduction to Knot Theory. Springer GTM 175, 1997.