CENTRAL UNIVERSITY OF HARYANA

End Semester Examinations April 2022

Programme: M.Sc. Mathematics **Session: 2021-22**

Semester: First **Max. Time: 3 Hours**

Course Title: Differential Equations **Max. Marks: 70**

Course Code: SPMMAT 01 01 04 C 3104

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) What do you mean by Lipschitz condition. Applying this definition, find whether or not the function $f(x, y) = y^{2/3}$ satisfy Lipchitz condition on the rectangle $R:\ |x| \leq 1, |y| \leq 1$.

b) Define the following terms with illustrations: Saddle point, Node and Spiral.

c) Find the solution of the equation $(D^2 - 1)y = 1$, which vanishes when $x = 0$ and tends to a finite limit as $x \to -\infty$. (D stands for d/dx)

d) Define Green's function. Also, explain the importance of Green's function.

e) Solve $py + qx + pq = 0$.

f) Classify $u_{xx} + u_{yy} + u_{zz} = 0$.

g) Solve $(D^2 + 3DD' + 2D'^2)z = x + y$.

Q 2. (2X7=14)

a) State and prove Picard's theorem on the existence of solutions of differential equations.

b) Prove that two solutions $y_1(x) and\ y_2(x)$ of the equation, $a_0(x)y'' + a_1(x)y'(x) + a_2(x)y = 0, a_0(x) \neq 0, x \in (a, b)$ are linearly dependent if and only if their Wronskian is identically zero.

c) Solve the differential equation $(8p^3 - 27)\ x = 12p^2y$ and investigate whether a singular solution exists.

Q3. (2X7=14)

a) Find the eigenvalues and eigenfunctions of the Strum-Liouville problem:
$X'' + \lambda X = 0, X'(0) = 0, X'(L) = 0$.

b) Find the series solution of $2x^2 \frac{d^2y}{dx^2} + x\ \frac{dy}{dx} + (x^2 - 3)y = 0$ near $x = 0$.

c) State Legendre's equation and find the solution of Legendre's equation about $x = 0$.

Q 4. (2X7=14)

a) Solve $z(x+2y)p - z(y+2x)q = y^2 - x^2$ by Lagrange's method.

b) Find the surface which is orthogonal to the one parameter system $z = cxy\,(x^2+y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$.

c) Find the complete integral of $xp + 3yq = 2\,(z - x^2q^2)$ by Charpit's method.

Q 5. (2X7=14)

a) Show that $\frac{1}{(bD-aD')^n}\, \emptyset(ax+by) = \frac{x^n}{b^n n!}\, \emptyset(ax+by)$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

b) Reduce the equation $(y-1)r - (y^2-1)s + y(y-1)t + p - q = 2ye^{2x}(1-y)^3 = 0$ to canonical form and hence solve it (where p, q, r, s and t have their usual meanings).

c) Solve one dimensional wave equation $\partial^2 y/\partial x^2 = (1/c^2)\,(\partial^2 y/\partial t^2)$. Deduce the expression for y satisfying the boundary conditions $y(0,t) = 0 = y(a,t)$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations April-2022

Programme	: Integrated B.Sc.-M.Sc. (Mathematics)	Session	: 2021-2022
Semester	: First	Max. Time	: 3 Hours
Course Title	: Calculus	Maximum Marks	: 70
Course Code	: SBSMAT 03 01 01 C 4046		

Instructions:

1. Question no. 1 has has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define hyperbolic functions with example. Show that $2sinh\ xcosh\ x = sinh\ 2x$. $(4\times3.5 = 14)$

   (b) If $y = sin(sinx)$, prove that $\frac{d^2y}{dx^2} + tanx\frac{dy}{dx} + ycos^2x = 0$.

   (c) To evaluate $\lim_{x\to\frac{\Pi}{4}} \frac{sinx-cosx}{x-\frac{\Pi}{4}}$.

   (d) Using a reduction formula to Evaluate $\int cos^3xdx$.

   (e) Plot the point P whose polar coordinates are $(-3, \frac{\Pi}{6})$. Find all the polar coordinates of P.

   (f) Find the velocity, speed and acceleration of a particle that moves along a curve in space described by the position vector $\overrightarrow{r(t)} = -3sin\ t\ \hat{i} + 3cos\ t\ \hat{j} + t^2\hat{k}, \quad 0 \le t \le 2\Pi$.

   (g) Define limit of vector functions. Find the limit of vector function $\overrightarrow{r(t)} = cos\ t\ \hat{i} + sin\ t\ \hat{j} + t\hat{k}$ at $t = \frac{\Pi}{4}$

2. (a) State and prove the Leibniz theorem for higher order derivatives for the product of two functions.

   (b) Let $Y = (x^2e^xcos\ x)$, then find Y_n. If $Y = x^2sin\ x$, then prove that

$$\frac{d^ny}{dx^n} = (x^2 - n^2 + n)sin(x + \frac{n\Pi}{2}) - 2nxcos(x + \frac{n\Pi}{2}).$$

   (c) Define concave up and concave down with examples. Find the intervals on which the function $f(x) = x^3 - 3x^2 + 3x - 3$ is concave up or concave down. Identify any inflection points. $(2\times7 = 14)$

3. (a) Define Cartesian and Polar coordinates with examples. Find the polar coordinates of the points with given cartesian coordinates P(-2,1)and vice versa.$(2 \times 7 = 14)$

   (b) State reduction Formulas. Use the reduction formula of $\int sin^nxcos^mxdx$ to evaluate $\int sin^2xcos^3x\ dx$.

   (c) Define volume by disk and washer method. The region bounded by the curve $y = \sqrt{x}$ the x-axis, and the line $x = 2$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

4. (a) Find the arc length of a circular helix given by:

$$\overrightarrow{r(t)} = Cost\hat{i} + Sint\hat{j} + t\hat{k}, \quad 0 \le t \le 2\Pi.$$

   Explain how to find an arc length parameterization?

   (b) Find the area of the surface obtained by revolving the parametric curve defined by $x = t^2$, $y = t^3$. $0 \le t \le 1$, about the y-axis.

(c) Describe the graph of the equation:

$$x^2 - 2x + 4y - 3 = 0.$$

Also find the axis of symmetric.

5. (a) Define differentiation of a vector function with example. Show that $\frac{d}{dt}[u(t) \times v(t)] = u'(t) \times v(t) + u(t) \times v'(t)$.

(b) Let $F(t) = t^2 i + tj - (sin\ t)k$ and $G(t) = ti + \frac{1}{t}j + 5k$. Find $(F+G)(t)$, $(F \times G)(t)$ and $(F.G)(t)$.

(c) Define continuity and Integration of vector functions with examples. For what values of t $F(t) = (sin\ t)i + (1-t)^{-1}j + (ln\ t)k$ is continuous.

CENTRAL UNIVERSITY OF HARYANA

End Semester Examination April 2022

Programm: M.Sc. Mathematics
Semester: First
Course Title: Complex Analysis
Course Code:SBSMAT01 01 03 C 3104

Session: 2021-22
Max. Time: 3 Hours
Max. Marks: 70

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

(1) (4× 3.5=14)

(a) Prove that $f(z) = |z|^2$ is differentiable but not analytic at $z = 0$.

(b) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

(c) Evaluate $\int_C (\bar{z})^2\, dz$ around the contour $|z-1| = 1$ in the counterclockwise sense.

(d) Evaluate $\int_C \frac{z+4}{z^2+2z+5}\, dz$, where C is the circle $|z+1| = 1$, traversed in the anticlockwise direction.

(e) Evaluate $\int_C \frac{\sin^6 z}{(z-\frac{\pi}{6})^2}\, dz$, where C is the circle $|z| = 1$, traversed in the anticlockwise direction.

(f) Expand the function $f(z) = \frac{z}{(z-1)(2-z)}$ in the Taylor series about the point $z = 0$, and also give the region of convergence.

(g) Evaluate $\int_C e^{-1/z} \sin(1/z)\, dz$, where C is the circle $|z| = 1$, traversed in the anticlockwise direction.

(2) (2× 7=14)

(a) Show that the function $f(z) = \sqrt{(|xy|)}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the origin.

(b) If $w(z) = \phi(x, y) + i\psi(x, y)$, represents the complex potential for an electric field and

$$\psi(x, y) = x^2 - y^2 + \frac{x}{x^2 + y^2},$$

then determine the function $\phi(x, y)$.

(c) Find the image of the circle $|z - a| = a$, where $a > 0$, under the transformation $w = 1/z$.

(3) (2× 7=14)

(a) State and prove Cauchy integral theorem.

(b) Find all possible Laurent series expansions of the function $f(z) = \frac{1}{z(z-2)}$ about the point $z = 0$, and also give the precise region of convergence in each case.

(c) Evaluate $\int_C \frac{(2 + 3\sin \pi z)}{z(z-1)^2} dz$, where C is the square having vertices at $3 + 3i,\ 3 - 3i,\ -3 + 3i,\ -3 - 3i$.

(4) (2× 7=14)

(a) Let γ be a closed contour in $\mathbb{C}$ and a be a complex number such that $a \notin \gamma$, prove that the index of γ about the point a, is an integer.

(b) Find the residue of $f(z) = \frac{z}{e^{-z^2} + 1}$ at the point $z = \infty$.

(c) Using calculus of residues, evaluate $\int_0^\infty \frac{\sin mx}{x(x^2+a^2)}dx,\ m>0,\ a>0.$

(5) (2× 7=14)

(a) Find the maximum modulus of the function $f(z) = \frac{(z-2)}{(z+2)}$ in the region $|z| \leq 1$.

(b) Using Rouche's theorem, find the number of roots of $z^7 - 4z^3 + z + 1 = 0$, which lie interior to the unit circle $|z| = 1$.

(c) State and prove Schwarz lemma.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations April-2022

Programme	: Integrated B.Sc.-M.Sc. (Mathematics)(Reappear)	Session	: 2022-2023
Semester	: First	Max. Time	: 3 Hours
Course Title	: Calculus(P)	Maximum Marks	: 70
Course Code	: SBSMAT 03 01 01 C 4046		

Instructions:

1. Question no. 1 has has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Find the length of a circle of radius r defined parametrically by $x = rcost$ and $y = rsint$, $0 \leq t \leq 2\pi$.
   (b) If $y = Sin(sinx)$, prove that $\frac{d^2y}{dx^2} + tanx\frac{dy}{dx} + ycos^2x = 0$.
   (c) Use L'Hospital rule to evaluate $\lim_{x \to \frac{\Pi}{4}} \frac{Sinx - cosx}{x - \frac{\Pi}{4}}$.
   (d) Using reduction formula to evaluate $\int Sin^3xdx$.
   (e) Plot the point P whose polar coordinates are $(2, \frac{\Pi}{4})$. Find all the polar coordinates of P.
   (f) Find the speed and acceleration of a particle that moves along a curve in space described by the position vector $\overrightarrow{r(t)} = -3Sint\hat{i} + 3Cost\hat{j} + t^2\hat{k}, \quad 0 \leq t \leq 2\Pi$.
   (g) Find the limit of vector function $\overrightarrow{r(t)} = 3Cost\hat{i} + Sint\hat{j} + 2t\hat{k}$ at $t = \frac{\Pi}{4}$.

2. (a) State and prove the Leibniz theorem for higher order derivatives for the product of two functions.
   (b) Let $Y = (x^2e^xCosx)$, then find Y_n. If $Y = x^2Sinx$, then prove that
   $$\frac{d^ny}{dx^n} = (x^2 - n^2 + n)Sin(x + \frac{n\Pi}{2}) - 2nxCos(x + \frac{n\Pi}{2}).$$
   (c) Find the intervals on which the function $f(x) = x^3 - 2x^2 + 6x - 4$ is concave up or concave down. Identify any inflection points.

3. (a) Define Cartesian and Polar coordinates with examples. Find the polar coordinates of the points with given cartesian coordinates P(-3,2) and vice versa.
   (b) Use the reduction formula of $\int Sin^nxcos^mxdx$ to evaluate $\int Sin^2xCos^3x$.
   (c) Define volume by washer method. The region bounded by the curve $y = \sqrt{x}$ the x-axis, and the line is revolved about the x-axis to generate a solid. Find the volume of the solid.

4. (a) Define an arc length parameterization. Find the arc length of a circular helix given by:
   $$\overrightarrow{r(t)} = Cost\hat{i} + Sint\hat{j} + t\hat{k}, \quad 0 \leq t \leq 2\Pi.$$
   (b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, \quad 1 \leq x \leq 2$, about the x-axis.
   (c) Describe the graph of the equation:
   $$x^2 - 3x + 5y - 7 = 0.$$
   Also find the axis of symmetric.

5. (a) Show that $\frac{d}{dt}[u(t) \times v(t)] = u^{'}(t) \times v(t) + u(t) \times v^{'}(t)$.

(b) Let $F(t) = t^3 i + tj - (Sint)k$ and $G(t) = ti + \frac{1}{t}j + 3k$. Find $(F+G)(t)$, $(F \times G)(t)$ and $(F.G)(t)$.

(c) Define continuity of vector functions give one example. For what values of t $F(t) = (Sint)i + (1-t)^{-1}j + (lnt)k$ is continuous.

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End Semester Examinations April 2022

Programme	: Integrated B.Sc.- M.Sc. Mathematics	Session	: 2021-2022
Semester	: First	Max. Time	: 3 Hours
Course Title	: Algebra	Max. Marks	: 70
Course Code	: SBSMAT 03 01 03 GE 5106		

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries threeand half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Express $\exp\left(\dfrac{x-a+iy}{x+a+iy}\right)$ into real and imaginary parts. (4x3.5=14)

   (b) Prove that the set of four vectors, $v_1=(1,0,-1)$, $v_2=(-1,0,0)$, $v_3=(1,0,1)$ and $v_4=(2,1,3)$ is linearly dependent.

   (c) Let $V=R^3=\{(x,y,z):x,y,z\in R\}$. Show that the subset of V, $W=\{(x,y,z):x-3y+4z=0,x,y,z\in R\}$ forms subspace of V.

   (d) State and prove Gauss Theorem.

   (e) Prove that the relation of congruency is an equivalence relation.

   (f) Solve the following system of equations

   $$x-y+z=1$$

   $$x+2y-z=0$$

   $$2x+y+3z=0\ .$$

   (g) Find (256, 1166) i.e. g.c.d. of 256 and 1166. Also express it as a linear combination of these two numbers.

2. (a) If α and β be the roots of $x^2-2x+4=0$, prove that $\alpha^n+\beta^n=2^{n+1}\cos\dfrac{n\pi}{3}$. (2x7=14)

   (b) Define the term one to one correspondence. Show that set **Z** of all integers is countably infinite set.

   (c) If $f:X\to Y$ be an everywhere defined invertible function and A and B be arbitrary non-empty subsets of Y. Show that (i) $f^{-1}(A\cup B)=f^{-1}(A)\cap f^{-1}(B)$ (ii) $f^{-1}(A\cap B)=f^{-1}(A)\cup f^{-1}(B)$

3. (a) State and prove Division algorithm. (2x7=14)

   (b) The linear congruence $ax\equiv b(\bmod m)$, where a is not congruent to 0 mod(m), has a solution if and only if (a,m) divides b.

   (c) State and prove Euclid's First Theorem.

4. (a) Reduce the matrix $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ to the normal form and hence determine its rank. (2x7=14)

(b) For what value of λ, does the system

$$-x+2y+z=1$$
$$3x-y+2z=1$$
$$y+\lambda z=1$$

has (i) no solution (ii) unique solution.

(c) Find p if the vector $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ p \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent.

5. (a) Find the eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. (2x7=14)

(b) Define Linear Transformation. Show that the function $T: R^2 \to R^3$ given by $T(x,y) = (x+y, x-y, y)$ is a linear transformation.

(c) Verify Cayley-Hamilton Theorem for the matrix A and compute A^{-1}, where $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$

CENTRAL UNIVERSITY OF HARYANA

End Semester Examinations April 2022

Programme: M.Sc. Mathematics **Session:** 2021-22

Semester: First **Max. Time:** 3 Hours

Course Title: Programming in C **Max. Marks: 70**

Course Code: SBSMAT 01 01 05 C 3104

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Write a short note on procedural language.

b) Which of the following are invalid variable names (identifiers) and why?

i) Ram_Krishan ii) continue iii) 3rd_row iv) n1+n2

c) Define logical operators and their precedence and associativity.

d) State and explain the errors in following input statements if variables are declared correctly.

i. scanf("%c %f",a,b);

ii. scanf("\n%d",a);

iii. scanf("%d %c %e",&a,&b,&c)

e) Consider the following program:

```
#include<stdio.h>
 void main() {
 int k=0;
for( ;; ) {
printf("%d", k++);
if(k>5)
  break; } }
```

What will be the result of attempting to compile and execute above program?

f) Differentiate between while and do - while statements.

g) Determine the output of the following program?

```
#include<stdio.h>
 void main( )
{          f( );
           f( );
           f(  );
 }
        void f(void)
        {          static int x=0;
                    printf("%d ", ++x);
         }
```

P.T.O.

Q 2. (2X7=14)

a) Explain each section in basic structure of C-programme.

b) Write down all primary data types, their syntax, and sizes and ranges on a 32 bit computer.

c) Define symbolic constants, their use, modifiability and various rules apply to #define statements for defining a symbolic constant.

Q3. (2X7=14)

a) Explain all the bitwise operators with examples, their associativity and precedence.

b) Write all steps and explain the output of the following program

```
#include<stdio.h>
main()
{
    int z;
    z=+4*5.0/2= =7&&(5!=3||3>=5)&&~0;
    printf("%d",z);
}
```

c) Explain *printf* format codes and format flags for outputs with examples.

Q 4. (2X7=14)

a) Write a short note on the following

i) *break* statement

ii) *for(; ;)* loop

iii) *else if* ladder

iv) *goto* statement

b) Write a C-program using do while loop to compute sum of first *n* terms of sin(x)

c) Use two-dimensional array to write a C- program for multiplication of two matrices A_{mxn} and B_{pxq}.

Q 5. (2X7=14)

a) Write a short note on the following

i) *static variable*

ii) *strcpy* ( ) function

iii) *return* statement

iv) * and & operators

b) Explain function definition, call and declaration with the help of a C- programme.

c) WAP using pointers to compute the sum of all elements stored in an array.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations April-2022

Programme	: M.Sc. Mathematics	Session	: 2021-2022
Semester	: First	Max. Time	: 3 Hours
Course Title	: Real Analysis	Maximum Marks	: 70
Course Code	: SPMMAT 01 01 01 C 3104		

Instructions:
1. Question no. 1 has has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Show that every convergent sequence is bounded but converse is not true justify your answer.
   (b) Define disconnected metric space and give an example of disconnected metric space.
   (c) Show that continuous image of a cauchy sequence is also a cauchy sequence.
   (d) Give an example of an ordered field which is not complete. Justify your answer.
   (e) Show that the Euclidean space is complete.
   (f) Define function of bounded variation and show that $\cos x$ is a function of bounded variation over a finite interval.
   (g) Define Riemann sum and state the conditions under which Riemann integral exist.

2. (a) Define completeness property of $\mathbb{R}$. State and prove the Archimedean property. If $S = \{\frac{1}{n} : n \in \mathbb{N}\}$, then show that $infS = 0$.
   (b) Define Lim_{Sup} and Lim_{inf}. of a sequence. Show that a convergent sequence in $\mathbb{R}$ can have $Lim_{Sup} = Lim_{inf}$. Show that $\lim \frac{1}{n} = 0$.
   (c) If $X = (x_n)$ and $Y = (y_n)$ be sequences of real number that converges to x and y respectively. Then show that the sequences $X+Y$ converges to $x+y$. and $X \times Y$ converges to $x \times y$ respectively. State whether the cvonverse is tre or not explain with an example.

3. (a) Define uniform convergence of sequence of functions. State and prove the Weierstrass M-test of sequence of functions.
   (b) Define pointwise convergence of a sequence of functions. Give an example to show that pointwise convergence does not imply uniform convergence.
   (c) Prove that a bounded function f is integrable on $[a, b]$ if and only if for each $\epsilon > 0$, there exist a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \epsilon$.

4. (a) Define bounded variation and total variation. Let f be of bounded variation on $[a, b]$. Let V be defined on $[a, b]$ as $V(x) = V_f(a, x)$, if $a < x \leq b$, $V(a) = 0$. Then show that V and $V - f$ are increasing function on $[a, b]$.
   (b) Suppose $f \geq 0$ is continuous on [a,b] and $\int_a^b f(x)dx = 0$. Then show that $f(x) = 0, \forall x \in [a, b]$. Also show that if $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$, then $c_1 f + c_2 g \in \mathcal{R}(\alpha)$ on $[a, b]$ (for any two constants c_1 and c_2) we have

$$\int_a^b (c_1 f + c_2 g) d_\alpha = c_1 \int_a^b f d_\alpha + c_2 \int_a^b g d_\alpha.$$

(c) State and prove implicit function Theorem.

5. (a) Define discrete metric space with example. Show that in a metric space (X, d) a sequence $\{x_n\}$ converges to p if and only if every subsequence x_{n_k} converges to p.

   (b) Prove that every compact subset F of a metric space (X, d) is closed.

   (c) State and prove Heine-Borel covering Theorem.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations April-2022

Programme	: M.Sc. Mathematics	Session	: 2021-22
Semester	: First	Max. Time	: 3 Hours
Course Title	: Algebra-I	Max. Marks	: 70
Course Code	: SBSMAT 01 01 02 C 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define permutation group. $(4 \times 3.5 = 14)$
   (b) Give an example of a non-abelian group each of whose subgroup is normal.
   (c) Find all the non-isomorphic abelian groups of order 8.
   (d) Find out all the conjugate classes of S_3.
   (e) Define a division ring and give an example of it.
   (f) Define nilpotent and idempotent element of a ring R. Find the idempotent and nilpotent elements in Z_6.
   (g) Show that the polynomial $x^2 + x + 2$ is irreducible over the field of integers modulo 3.

2. (a) State and prove Cayley's Theorem in group theory. $2 \times 7 = 14$
   (b) If H and K are two subgroups of a group G and H is normal in G then show that $\frac{HK}{H} \approx \frac{K}{H \cap K}$.
   (c) If G be finite cyclic group of order n, prove that to each divisor m of n, there exists a unique subgroup of order m.

3. (a) Show that a group of order p^2q (p and q are primes) is not simple. $2 \times 7 = 14$
   (b) State and prove Cauchy's theorem.
   (c) Prove that a group of order p^n must have a non-trivial centre where (p is primes).

4. (a) State and prove the Fundamental Theorem of ring homomorphisms. $2 \times 7 = 14$
   (b) Let R be a commutative ring with unity. Prove that an ideal P of R is prime if and only if $\frac{R}{P}$ is an integral domain.
   (c) Show that a finite integral domain is a filed. Give an example of an infinite integral domain which is not field.

5. (a) Prove that in a Principle Integral Domain (PID) an element is prime if and only if it is irreducible. $2 \times 7 = 14$
   (b) Show that the ring of Gaussian integers $Z[i]$ is a Euclidean domain.
   (c) State and prove Einstein Criterion for irreducibility of a polynomial with integer coefficients over the filed Q of rational numbers.