# CENTRAL UNIVERSITY OF HARYANA
## Second Semester Term End Examinations September 2022

| | | | |
|---|---|---|---|
| Programme | : M.Sc. Mathematics | Session | : 2021-2022 |
| Semester | : Second | Max. Time | : 3 Hours |
| Course Title | : Wavelet Analysis | Maximum Marks | : 70 |
| Course Code | : SBSMAT 01 02 01 DCEC 3104 | | |

**Instructions:**
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts each and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define Hilbert space, give an example. $(4 \times 3.5 = 14)$

   (b) Let $\{e_k\}_{k=1}^{\infty}$ be an orthonormal basis for $H$. Show that $\{f_k\}_{k=1}^{\infty} = \{e_1, e_1, e_2, e_3, ...\}$ is a frame with frame bounds A=1, B=2.

   (c) Define frame, give an example of tight frame.

   (d) Define Multiresolution Analysis. Show that the function $\phi(x-k)$ has unit norm in $L^2$.

   (e) Draw the graph of the function $f(x) = 2\phi(x) + 3\phi(x-1) + 3\phi(x-2) - \phi(x-3) \in V_0$ and also find all the discontinuities of $f$ at $x$.

   (f) Write four main difference between the Haar Wavelet and the Daubechies wavelet.

   (g) Write down the majour comparision between continuous Fourier transform and continuous wavelet transform.

2. (a) Define Bessel sequence. Let $\{e_1, e_2, ..., e_n\}$ be a finite orthonormal set in an inner product space $X$. Then, for any $x \in X$, show that $\sum_{i=1}^{n} |\langle x, e_i\rangle|^2 \leq \|x\|^2$. $(7 \times 2 = 14)$

   (b) Let $\{f_k\}_{k=1}^{\infty}$ be a sequence in $H$ and $B > 0$ be given. Then, $\{f_k\}_{k=1}^{\infty}$ is a Bessel sequence with Bessel bound $B$ if and only if $T : \{f_k\}_{k=1}^{\infty} \longrightarrow \sum_{k=1}^{\infty} c_k f_k$ defines a bounded linear operator from $l^2(\mathbb{N})$ into $H$ and $\|T\| \leq \sqrt{B}$.

   (c) Let $\{f_k\}_{k=1}^{\infty} = \{Ue_k\}_{k=1}^{\infty}$ be a Riesz basis for $H$, and let G from $l^2(\mathbb{N}) \longrightarrow l^2(\mathbb{N})$ be the Gram matrix. Then the optimal Riesz bounds are $A = \frac{1}{\|U^{-1}\|^2} = \frac{1}{\|G^{-1}\|}$ and $B = \|U\|^2 = \|G\|$.

3. (a) For a sequence $\{f_k\}_{k=1}^{\infty}$ in $H$. Then, $\{f_k\}_{k=1}^{\infty}$ is a Riesz basis for $H$ if and only if $\{f_k\}_{k=1}^{\infty}$ is complete in $H$, and there exists $A, B > 0$ such that for every finite scalar sequence $\{c_k\}$, one has $A\sum |c_k|^2 \leq \|\sum c_k f_k\|^2 \leq B\sum |c_k|^2$. $(7 \times 2 = 14)$

   (b) Let $\{f_k\}_{k=1}^{\infty}$ be a frame with frame operator S. Then, show that $f = \sum_{k=1} \infty\langle f, S^{-1}f_k,\rangle, \ \forall f \in H$, and the series converges unconditionally for all $f \in H$.

   (c) Let $\{f_k\}_{k=1}^{\infty}$ be a frame with frame operator S and frame bounds $A$, $B$. Then, show that $\{S^{-1}f_k\}_{k=1}^{\infty}$ is a frame with frame operator $S^{-1}$ and frame bounds $B^{-1}, A^{-1}$.

4. (a) Show that a function $f(x)$ belongs to $V_0$ if and only if $f(2^j x)$ belongs to $V_j$. $(7 \times 2 = 14)$

   (b) Let $W_j$ be the space of functions of the form $\sum_{k\in\mathbb{Z}} a_k \psi(2^j x - k)$; $a_k \in \mathbb{R}$, where, we assume that only finite number of $a_k$ are non-zero. Then, show that $W_j$ is the orthogonal complement of $V_j$ in $V_{j+1}$ and $V_{j+1} = V_j \oplus W_j$.

(c) Suppose $\{V_j; j \in \mathbb{Z}\}$ is a multiresolution analysis with scaling function $\phi$. Then, show that for any $j \in \mathbb{Z}$, the set of the functions $\{\phi_{jk}(x) = 2^{\frac{j}{2}}\phi(2^j x - k),\ k \in \mathbb{Z}\}$ is an orthonormal basis for $V_j$.

5. (a) If $\psi$ is a wavelet and $\phi$ is a bounded integrable function, then prove that convolution function $\psi * \phi$ is a wavelet. (7 × 2 = 14)

(b) For all $f, g \in L^2(\mathbb{R})$, prove that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dadb}{a^2}(T^{wav}f)(a,b)\overline{(T^{wav}g)(a,b)} = c\psi\langle f, g\rangle.$$

(c) Let $\omega \in L^2(\mathbb{R})$ be choosen such that both $\omega$ and its Fourier transform $\hat{\omega}$ satisfy $t\omega(t) \in L^2(\mathbb{R})$. Then

$$\Delta\omega\Delta\hat{\omega} \geq \frac{1}{2}.$$

Furthermore, equality is attained if and only if $\omega(t) = ce^{iat}g_\alpha(t-b)$, where $c \neq 0$, $\alpha > 0$, and $a, b \in \mathbb{R}$.

# CENTRAL UNIVERSITY OF HARYANA

**Second Semester Term End Examinations August- September 2022**

**Programme: Integrated BSc-MSc (Mathematics)** **Session: 2021-22**

**Semester: II** **Max. Time: 3 Hours**

**Course Title: Differential Equations** **Max. Marks: 70**

**Course Code: SBSMAT 03 02 02 C 4046**

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**Instructions:**

*1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.*
*2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.*

Q 1(a). Test the exactness and solve: $(1 + 2xy \cos x^2 - 2xy)\, dx + (\sin x^2 - x^2)\, dy = 0$

Q 1(b). Find the integrating factor for the equation: $y(xy + 2x^2y^3)\, dx + x(xy - x^2y^2)\, dy = 0$

Q 1(c). State the Compartmental Model and the Balance Law.

Q 1(d). Solve the homogeneous equation: $\frac{d^4x}{dt^4} + 4x = 0$

Q 1(e). Find the Particular Integral of the equation: $(D + 2)(D - 1)^2 y = 2 \sinh x$

Q 1(f) Describe epidemic model of influenza.

Q 1(g). Solve the Cauchy-Euler equation: $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$

Q 2(a). Solve the Bernoulli's equation: $\frac{dx}{dy} - yx = y^3x^2$

Q 2(b). Show that the differential equation $\mathbf{M}(x,y)dx + \mathbf{N}(x,y)\, dy = 0$ is exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Q 2(c). Solve the initial value problem: $(x^2 + 1)\frac{dy}{dx} + 4xy = x$;  $y(2)=1$.

Q 3(a). Develop a very simple mathematical model which describes the exponential growth of a population. Write the model assumptions, governing differential equation and initial condition. Find the solution, and an expression for the time for the population to double in size.

Q 3(b). Solve the differential equation along with the initial condition for the concentration C of the pollutant in the lake, $\frac{dC}{dt} = \frac{F}{V}C_{in} - \frac{F}{V}C$, $C(0) = c_o$

How long will it take for the lake's pollution level to reach 5% of its initial level, if only fresh water flows into the lake?

Q 3(c). Let x(t) be the amount of a drug in the GI-tract at time t and y(t) the amount in the bloodstream at time t. Model the case of drug assimilation for a course of cold pills. Write down the governing equations along with model assumptions. Find the solution of resulting sequence of linear equations.

Q 4(a). Solve the equation: $(D-2)^2 y = 8\,(e^{2x} + \sin 2x + x^2)$

Q 4(b). Find the complete solution of $y'' - 2y' + 2y = x + e^x \cos x$

Q 4(c). Using the properties of Wronskian, show that the functions $e^x$, $e^{-x}$, and $e^{2x}$ are linearly independent on every real interval.

Q 5(a). Using method of Undetermined Coefficients, solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$

Q 5(b). Using method of Variation of Parameters, find the particular integral of

$$\frac{d^4y}{dx^4} + y = \tan x$$

Q 5(c). Consider the Lotka-Volterra predator-prey model, assuming X and Y denote respectively the prey and predator population, $\frac{dX}{dt} = \beta_1 X - c_1 XY,\quad \frac{dY}{dt} = -\alpha_1 X + c_2 XY.$

Here, $c_1$ *and* $c_2$ are interaction parameters, and $\beta_1$ is prey per-capita birth rate and $\alpha_1$ is the predator per-capita death rate. Find the equilibrium points for the system. Draw the direction vector diagram for the phase-plane.

# CENTRAL UNIVERSITY OF HARYANA

**Second Semester Term End Examinations August-September 2022**

**Programme: M. Sc. (Mathematics)** | **Session: 2021-22**

**Semester: Second** | **Max. Time: 3 Hours**

**Course Title: Numerical Methods** | **Max. Marks: 70**

**Course Code: SBSMAT 01 02 02 GEC 2124**

**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) An approximation to the value of $\pi$ is given by $\frac{22}{7}$, while its true value is 3.1415926. Calculate the absolute, relative and percentage errors in the approximation.

b) How you will decompose a square matrix of order 3 in Crout method. Illustrate it with an example.

c) What is Lagrange's Interpolation formula?

d) Prove that $\mu^2 \equiv 1 + \frac{1}{4}\delta^2$.

e) Solve the integral $\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^2}} dx$ numerically with Gauss-Chebyshev 2-points formula.

f) Discuss in brief about cubic spline interpolation.

g) Calculate value of y (0.3) for following IVP

$\frac{dy}{dx} = x + y, y(0) = 1$ by Euler method with step size h=0.1.

Q 2. (2X7=14)

a) Use Regula-Falsi method to compute the root of the equation $x^3 - 4x - 9 = 0$ in the interval (2, 3) correct to five decimal places.

b) What do you mean by order of convergence. Derive an expression for order of convergence in case of Newton-Raphson method.

c) Solve the following system of linear equations corrects up to three decimal places using the Gauss-Seidal method:

$3x_1 + x_2 - x_3 = 3, 2x_1 + 4x_2 + x_3 = 7, x_1 - x_2 + 4x_3 = 4$

Q3. (2X7=14)

a) Use Newton backward difference formula to find the value of f(1.9) for the following data set:

| | | | | | |
|---|---|---|---|---|---|
| x: | 1 | 1.25 | 1.5 | 1.75 | 2.0 |
| f(x) | 0 | 0.223144 | 0.405465 | 0.559616 | 0.693147 |

b) Use Stirling formula to find the value of $y_{18}$ for the given data set:

$y_{10} = 362, y_{15} = 487, y_{20} = 642, y_{25} = 897, y_{30} = 1206$

c) Use Newton divided difference formula to derive interpolating polynomial for the data points (0, -1), (1, 1), (2, 9), (3, 29), (5, 129) and hence compute the value of y(4).

Q 4. (2X7=14)

a) Find the first and second derivatives of y=f(x) at $x = 0$ *and* 0.3 from the following table.

| | | | | |
|---|---|---|---|---|
| x | 0 | 0.1 | 0.2 | 0.3 |
| y | 1 | 1.105170 | 1.221403 | 1.349859 |

b) Evaluate the integral $\int_1^{2.2} \frac{1}{1+2x+x^2} dx$ by Trapezoidal and Simpson rules by dividing the interval into 12 equal subintervals.

c) Fit a quadratic curve to the following data and compute the value of variable y at point $x = 3$

| | | | | | |
|---|---|---|---|---|---|
| x: | 0 | 1 | 2 | 4 | 5 |
| y: | -2 | 0 | 10 | 78 | 148 |

Q 5. (2X7=14)

a) Use Runge-Kutta fourth order method with step size h=0.1 for the IVP $\frac{dy}{dx} = x - y^2, y(1) = 2$ to compute y (1.2).

b) Use Milne predictor-corrector method to compute y (0.4) from differential equation $\frac{dy}{dx} = y^2 - x^2$ and following values

| | | | | |
|---|---|---|---|---|
| x: | 0 | 0.1 | 0.2 | 0.3 |
| y: | 1 | 1.11 | 1.25 | 1.42 |

c) Solve the differential equation $(1-x)y'' + xy' - y = (1-x)^2;\ 0 \le x \le 1$ subject to boundary conditions $y(0) = 1, y(1) = 3$ by taking four equal subintervals of step size h=0.25. Use central difference approximations.

# CENTRAL UNIVERSITY OF HARYANA
## Second Semester Term End Examinations September-2022

| | | | |
|---|---|---|---|
| Programme | : Integrated B.Sc.-M.Sc. Mathematics | Session | : 2021-22 |
| Semester | : Second | Max. Time | : 3 Hours |
| Course Title | : Real Analysis | Max. Marks | : 70 |
| Course Code | : SBSMAT 03 02 01 C 5106 | | |

**Instructions:**
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define interior point with example. $(4 \times 3.5 = 14)$
   (b) Prove that union of two open sets is open set.
   (c) Give an example of an infinite set having no limit point.
   (d) Prove that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{1/n} = e$.
   (e) State Cauchy first theorem on limit.
   (f) State Cauchy convergence criterion for an infinite series.
   (g) Test for convergence of $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$.

2. (a) State and prove Archimedian property of real numbers. $(7 \times 2 = 14)$
   (b) Show that the set $S = \{x : x \in Q^+ \text{and } 0 < x^2 < 3\}$ does not have any least upper bound in $Q$.
   (c) Prove that the set of real numbers is uncountable.

3. (a) Prove that every bounded and infinite set has a limit point. $(7 \times 2 = 14)$
   (b) Show that every convergent sequence has a unique limit point.
   (c) Prove that the only limit point of the set $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is 0.

4. (a) State and prove Cauchy second theorem on limits. Show that $\lim_{n\to\infty} \left(\frac{(3n)!}{(n!)^3}\right)^{1/n} = 27$. $(7\times 2 = 14)$
   (b) Prove that $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ exists and lies between 2 and 3.
   (c) Prove that every bounded sequence has a limit point.

5. (a) Test the convergence or divergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{3^n n^2}, x > 0$. $(7 \times 2 = 14)$
   (b) Using integral test, show that the series $\sum_{n=0}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$.
   (c) Prove that every absolutely convergent series is convergent. Test the absolute convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \sin nx}{n^3}$.

# CENTRAL UNIVERSITY OF HARYANA

**Second Semester Term End Examinations August-September 2022**

**Programme: B.Sc. - M.Sc. (Integrated)** **Session: 2021-22**

**Semester: Second** **Max. Time: 3 Hours**

**Course Title: Real analysis** **Max. Marks: 70**

**Course Code: SBSMAT 03 02 03 GE 5106**

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Define Supremum and Infimum of a set. Find supremum and infimum of set

$$S = \left\{ \frac{3n+2}{2n+1} : n \in \mathrm{N} \right\}.$$

b) Show that the union of a finite number of closed sets is a closed set.

c) Find the derived set of

i. $\left\{ \frac{1}{m} + \frac{1}{n}; m \in \mathrm{N}, n \in \mathrm{N} \right\}$

ii. $\left\{ (-1)^n + \frac{1}{n}; n \in \mathrm{N} \right\}$

iii. $\left\{ 2^n + \frac{1}{2^n}; n \in \mathrm{N} \right\}$

d) Explain monotone sequence with example.

e) If $S_{n+1} = \sqrt{7S_n}$ , $S_1 = 1$, prove that $\langle S_n \rangle$ is convergent. What is its limit?

f) Discuss the convergence of $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \ldots$ .

g) Explain Leibnitz's Test.

Q 2. (2X7=14)

a) State and prove Archimedian property of real numbers.

b) Define Countable and Uncountable set. Show that the unit interval [0,1] is not countable.

c) Prove that every non-empty set of real numbers, which is bounded above, has the least upper bound.

Q3. (2X7=14)

a) Prove that between any two distinct real numbers, there are infinitely many rational numbers. Also, prove that between any two distinct real numbers, there are infinitely many irrational numbers.

b) Define an open set and prove that the union of an arbitrary family of open sets is open. Give an example to show that the intersection of an arbitrary family of open sets may not be open.

c) State and prove Bolzano-Weierstrass Theorem on limit points.

Q 4. (2X7=14)

a) Show that if $\langle a_n \rangle$ converges to $l$, then the sequence $\langle x_n \rangle$ where

$x_n = \dfrac{a_1 + a_2 + ... + a_n}{n}$ also converges to $l$. Using this theorem, show that

$$\lim_{n\to\infty} \frac{1}{n}\left(\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + ..... + \frac{n+1}{n}\right) = 1$$

b) Prove that every monotonically increasing sequence which is bounded above converge to its least upper bound. Also, prove that the necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

c) State and prove Cauchy's general principle of convergence. Prove that the sequence $\langle a_n \rangle$, where $a_n = 8 + \dfrac{1}{n^3}$ is a Cauchy sequence.

Q 5. (2X7=14)

a) Define Absolute and Conditional Convergence. Test the convergence and absolute convergence of the series: $1 - \dfrac{1}{2^3}(1+2) + \dfrac{1}{3^3}(1+2+3) - .....$

b) Discuss the convergence of the series:

i. $\dfrac{1}{2\sqrt{1}} + \dfrac{x^2}{3\sqrt{2}} + \dfrac{x^4}{4\sqrt{3}} + \dfrac{x^6}{5\sqrt{4}} + ......$ $(x > 0)$

ii. $\dfrac{1^3}{3} + \dfrac{2^3}{3^2} + 1 + \dfrac{4^3}{3^4} + .......$

c) Test the convergence of the series:

i. $\sum\left(1 - \cos\dfrac{\pi}{n}\right)$

ii. $\sum_{n=1}^{\infty} n e^{-n^2}$

# CENTRAL UNIVERSITY OF HARYANA

**Second Semester Term End Examinations August-September 2022**

**Programme: M.Sc Mathematics** | **Session: 2021-22**

**Semester: II** | **Max. Time: 3 Hours**

**Course Title: Linear Algebra** | **Max. Marks: 70**

**Course Code: SBSMAT 01 02 01 C 3104**

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Let $u = (1,-3,1,-2)$, find $\|u\|_1, \|u\|_2, \|u\|_\infty$.

b) Let $u_1$ and $u_2$ are nonzero orthogonal vectors and $w$ is any vectors in $V$. Find $a$ and $b$ so that $w'$ is orthogonal to $u_1$ and $u_2$, $w' = w + au_1 + bu_2$.

c) Prove that $T: R^2 \to R^2$, $T(x,y) = (x+y, 2x-3y)$ and $F: R^2 \to R^2$, $G(x,y) = (x-2y, 3x-5y)$ are similar.

d) Let $V = R^3$ and $W = \{(x_1, x_2, x_3) | x_1 = 0\}$ be a subspaces of $V$. Find of $\dim(V/W)$.

e) State the primary decomposition theorem.

f) Let $X$ be the subset of vector space $V$, prove that $X^\perp = span\,(X)^\perp$.

g) Suppose $T_1$ and $T_2$ are nilpotent operators that commutes, prove that $T_1T_2$ and $T_1 + T_2$ are also nilpotent.

Q 2. (2X7=14)

a) (i) Let $T: V_3 \to V_3$ such that $T(1,1,3) = (1,2,3)$ and $T(1,0,1) = (5,5,1)$. Determine there exists a linear map or not, if exists find the general formula.

(ii) Let $W = \{(x_1, x_2, \ldots x_n) \in R^n \,/\, a_1x_1 + a_2x_2 + \ldots + a_nx_n = 0\}$. Prove that $W$ is a subspace of $R^n$ also find dim of $W$.

b) Let $V = P_3$, the space of all real polynomials of degree atmost 3, $U$ and $W$ are two subspaces of $V$, where $U = \{p \in V \,/\, p(2) = 0\}$ and $W = \{p \in V \,/\, p'(2) = 0\}$, find basis and dimensions of $U, W, U \cap W$ and $U + W$.

c) (i) Find one vector in $R^3$ that spans the intersection of $U$ = {xy-plane} and $W = [(1,1,1), (1,3,5)]$.

(ii) Let $T: U \to V$ be a nonsingular linear mapping. Prove that the image of any linearly independent set is linearly independent.

Q3. (2X7=14)

a) Let $T : R^3 \to R^2$, $T(x, y, z) = (2x + 5y - 3z, x - 4y - 3z)$ and $F : R^3 \to R^2$ defined by $F(u) = [T]u$. Find $[T]$ and $[F]$ relative to the basis $S = \{(1,1,1), (1,1,0), (-1,0,-1)$ and $f = \{(1,1), (1,-1)\}$.

b) Whether the matrix $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 8 & 3 \end{bmatrix}$ is diagonalizable? If yes, find a diagonal matrix which is similar to $A$.

c) Find the characteristics and minimal polynomials of the matrix

$$\begin{bmatrix} 1 & 0 & -2 & 2 & 4 \\ -3 & 2 & 0 & 1 & 3 \\ 0 & 3 & -4 & 1 & 1 \\ 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}.$$

Q 4. (2X7=14)

a) (i) Let $R^7$ be the vector space over the rational field $Q$ and let $T$ be a linear operator on $V$ whose minimal polynomial is $m(t) = (t^2 + 7t + 2)(t - 1)^2$. Find the all possible rational canonical forms.

(ii) Define the Dual Space and Dual basis of a vector space.

b) Find an orthogonal change of coordinates $X = PY$ that diagonalizes of the following quadratic forms $q(x, y, z) = 2x^2 - 4xy + 5y^2 + 2xz - 4yz + 2z^2$. Find the corresponding diagonal quadratic form $q(Y)$.

c) (i) Find a basis of the annihilator of $W^0$ of $W = [(1, -1,0,3), (-1,0,1,1)]$.

(ii) Find the matrix representation of the bilinear form $f((x, y), (s, t)) = 2xs - 3xt + 2ys + 1yt$ in the basis $\{(1, -11), (3,7)\}$.

Q 5. (2X7=14)

a) (i) Let $\lambda$ be an eigenvalue of a linear operator T on V. Prove that if $T^* = -T$ then $\lambda$ is pure imaginary.

(ii) Suppose $A_1$ and $A_2$ are two subsets of $V$ .Prove that if $A_1 \subseteq A_2$ then $A_2^\perp \subseteq A_1^\perp$, where $A_1^\perp$ and $A_2^\perp$ are orthogonal complement of $A_1$ and $A_2$ respectively.

b) (i) Prove that if $S$ is orthogonal set of nonzero vectors then $S$ is linearly independent.

(ii) Let $S$ be a subspace of the vector space $V$. Prove that $= S \oplus S^\perp$, where $S^\perp$ is the orthogonal compliment of $S$.

c) (i) Show any operator $T$ is the sum of a self-adjoint operator and a skew-adjoint operator.

(ii) Let $S = \{(1,1,1), (1,2, -1), (0,0,1)\}$ whether $S$ is linearly independent or not? If yes, find the orthonormal set corresponding to $S$.

# CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Semester Examinations August-September 2022

**Programme: M. Sc. (Mathematics)** **Session: 2021-22**

**Semester: Second** **Max. Time: 3 Hours**

**Course Title: Topology** **Max. Marks: 70**

**Course Code:** SBS MAT 01 02 02 C 3104

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) If $X$ is any set, prove that the collection of all one-point subsets of $X$ is a basis for the discrete topology on X.

b) Show that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in $X \times X$.

c) Let $\pi_1 : X \times Y \to X$ be defined by the equation $\pi_1(x, y) = x$; and $\pi_2 : X \times Y \to Y$ be defined by the equation $\pi_2(x, y) = y$. Prove that the maps $\pi_1$ and $\pi_2$ are open maps.

d) Let $S^1 = \{x \times y : x^2 + y^2 = 1\}$ be the unit circle considered as a subspace of the plane $\mathbb{R}^2$, and let $F : [0,1) \to S^1$ be the map defined by $F(t) = (cos2\pi t, sin2\pi t)$. Check whether it is homeomorphism or not.

e) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

f) Prove that every compact subspace of a Hausdorff space is closed.

g) Prove that the one-point compactification of $\mathbb{R}^2$ is homeomorphic to the sphere $S^2$.

Q 2. (2X7=14)

a) Prove that the topologies $\mathbb{R}_l$ and $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one and other.

b) Define Hausdorff spaces and prove that every finite point set in a Hausdorff space $X$ is closed.

c) Define limit point in a topological space. Let $A$ be a subset of the topological space $X$, and $A'$ be the set of all limit points of $A$, then prove that $\bar{A} = A \cup A'$ .

Q3. (2X7=14)

a) Let $X$ be an ordered set in the order topology and $Y$ be a subset of $X$ that is convex in $X$, then prove that the bordered topology on $Y$ is the same as the topology $Y$ inherits as a subspace of $X$.

b) Let $f: A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Prove that the function $f$ is continuous if and only if the functions $f_1: A \to X$ and $f_2: A \to Y$ are continuous.

c) State and prove Tychonoff theorem.

Q 4. (2X7=14)

a) Prove that the Topologist's sine curve in $\mathbb{R}^2$ is connected but not path connected.

b) State and prove Tube lemma.

c) Prove that every closed interval in $\mathbb{R}$ is compact.

Q 5. (2X7=14)

a) Prove that the product of two Lindelöff spaces need not be Lindelöff.

b) Prove that every regular space with a countable basis is normal.

c) State and prove Tietze extension theorem.

**CENTRAL UNIVERSITY OF HARYANA**

**Second Semester Term End Examinations August-September 2022**

**Programme: M.Sc. Mathematics** **Session: 2021-22**

**Semester: Second** **Max. Time: 3 Hours**

**Course Title: Typesetting in LaTeX** **Max. Marks: 70**

**Course Code: SBSMAT 01 02 05 C 2023**

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5)

a) What are the various spacing commands in math mode?

b) Write a code in LaTeX to place an equation in a box with a tag within the box, and indicate which package is necessary for the command to function.

c) How textwidth and textheight settings affect margins.

d) Describe the various tags in LaTeX for formatting text.

e) Name the package which is used to put header and footer in a LaTeX document and show its usage.

f) What package(s) do you require to wrap a figure around the text? Include an example as well.

g) Give the command using PStricks to draw an elliptic arc having vertical radius 2 cm and horizontal radius 5 cm.

Q 2. (2X7=14)

a) How can you create an ordered or unordered list? Provide examples for each. Also prepare a nested list. Write a code in LaTeX for:

(i) One

(ii) Two

(iii) Three

1) One

2) Two

3) Three.

b) Write a code in LaTeX for

$$\begin{aligned} x &= y & w &= z & a &= b+c \\ 2x &= -y & 3w &= -2z & a &= b \\ -4+5x &= 2+y & w+2 &= -1+w & ab &= 3d+4. \end{aligned}$$

c) Correct the following input as per LaTeX commands: If \$x = α and \$ y = β \$ then \$ frac{α}β} =2 \$ and in PStricks, what PS stands for?

Q3. (2X7=14)

a) Write the code in LaTeX to plot the curves $y = sin\ 2x$ and $y = cos\ x$ on the same coordinate system for x∈ [0,2π]. Show the sine function as a solid curve and cosine function as a dashed curve.

b) What is the difference between the following environments in LaTeX?

(i) \vdots and \ddots

(ii) eqnarray and eqnarray*

(iii) enumerate and itemize.

c) Write a code in LaTeX for typesetting the following expression: $\frac{\frac{5}{ab^2}-\frac{2}{a^2b}}{\frac{3}{a^2b^2}+\frac{4}{ab}} = \frac{\frac{5b-2a}{ab^2+2ab}}{\frac{3+4ab}{a^2b^2}}$.

Q 4. (2X7=14)

a) Find the errors in the following LaTeX commands, write the corrected version and its output.

```
\Documentclass{beamer}
 \usetheme{CambridgeUS}
\begin{title}
{SYSTEM OF LINEAR EQUATIONS}
\end{title}
 \author{XYZ}
\begin{document}
\maketitle
 \begin{frame}
\frametitle{System of Linear Equations}
\begin{eqnarray*}
a_{11}x_1+a_{12}x_2+\cdots + a_{1n}x_n = b_1 \\
a_{21}x_1+a_{22}x_2+\cdots + a_{2n}x_n = b_2 \newline
\vdots \vdots \ddots \vdots & & \vdots \newline
a_{m1}x_1+a_{m2}x_2+\cdots + a_{mn}x_n = b_m
\end{ eqnarray*}
 In the matrix form it can be written as \textbf{AX = b}. The augmented matrix of the system is
\begin{equation}
 M=[A|b]=\left[\begin{matrix}{cccc|c}
a_{11} & a_{12} & \cdots &a_{1n} & b_1\\
a_{21} & a_{22} & \cdots &a_{2n} & b_2\\
\vdots & \vdots &\ddots &\vdots & \vdots\\
a_{m1} & a_{m2} & \cdots & a_{mn} & b_m\\
\end{matrix}\right]
\end{equation}
```

```
\begin{frame}

\frametitle{System of Linear Equations}

The system of linear equations is consistent if rank of [A|b] is equal to the rank of $A$ otherwise
inconsistent.

\end{frame}

\begin{frame}

\start{center}

\Huge{Thank You}

\end{frame}

\end{center}
```

(i) Write the command in LaTeX to generate the expression $y^{y^{y^y}}$.

(ii) What is the output of the command \psarc(1 1) {2} {0} {30} in PS tricks.

c)

(i) ..................... command is used to create horizontal dots above the line in LaTeX.

(ii) The string {c c c} is used to define ............................ and ................................ in the array environment in LaTeX.

(iii) Write the LaTex command for the following:

$$|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0. \end{cases}$$

(iv) Write the LaTex command for the following:

$$W = S_{n_m} + \cfrac{1}{S_{n_{m-1}} + \cfrac{1}{\ddots + \cfrac{1}{S_{n_1}}}}$$

Q 5. (2X7=14)

a) Write a code in Latex to typeset the following:

A system of linear equations in $n$ variables $x_1, x_2, x_3 \ldots x_n$ can be represented as

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}.$$

b) Write the input command in LaTeX to produce the following:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right).$$

c) Create the following presentation in LaTeX using beamer:

# Your Presentation

You

Central University of Haryana

September 2022



## Introduction

- Your introduction goes here!
- Use `itemize` to organize your main points.

**Examples**

Some examples of commonly used commands and features are included, to help you get started.

## Tables

| Item | Quantity |
|---|---|
| Widgets | 42 |
| Gadgets | 13 |

Table 1: An example table.

# Readable Mathematics

Let $X_1, X_2, \ldots, X_n$ be a sequence of independent and identically distributed random variables with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n} \sum_{i}^{n} X_i$$

denote their mean.

# CENTRAL UNIVERSITY OF HARYANA

**Second Semester Term End Examinations August-September 2022**

**Programme: M.Sc. Mathematics** | **Session: 2021-22**

**Semester: Second Semester** | **Max. Time: 3 Hours**

**Course Title:** Numerical Analysis | **Max. Marks: 70**

**Course Code:** SBSMAT 01 02 03 C 3104

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Compute the absolute error and relative error in the four significant digits rounding approximation of the number 64.2685.

b) In Bisection algorithm, let *M* denotes the length of interval [*a*, *b*]. Let $\{x_0, x_1, ...\}$ represents successive midpoints generated by Bisection method. Show that, $|x_{i+1} - x_i| = M / 2^{i+2}$.

c) Use the definitions of various finite operators to prove that $E^{1/2} \equiv \mu + \frac{\delta}{2}$.

d) Approximate integral $\int_2^4 \frac{e^x - \cos(x)}{x^2} dx$ with the help of Gauss-Chebyshev 2–points formula.

**e)** Approximate the value $f'(1.2)$ for the following data set.

| $x$: | 1 | 1.25 | 1.5 | 1.75 | 2.0 |
|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.223144 | 0.405465 | 0.559616 | 0.693147 |

f) Use Taylor series of order four to estimate $y(0.1)$ for the IVP $y' = x^2 y - 1,\ y(0) = 1$.

g) Compute the step size *h* for the IVP $\frac{dy}{dx} = -20y,\ \ y(0) = 1$, such that Euler method is stable.

Q 2. (2X7=14)

a)

(i) The bacteria concentration (*C*) in a reservoir varies as $C = 3e^{-1.4t} + e^{-0.2t}$. Using Newton-Raphson method, calculate the time required for the bacteria concentration to be 0.7 correct up to 3 decimal places.

(ii) For the function $x - e^{-x} = 0$ locate an interval containing the smallest positive zero and show that four conditions for the convergence of Newton-Raphson and Secant methods are satisfied.

b)

Solve the following system of linear equations by Gauss elimination method with partial pivoting using three significant digits floating points rounding arithmetic.

$$2x_1 + x_2 + 2x_3 = 4$$
$$2x_1 - x_2 + 100x_3 = 100$$
$$3x_1 + 50x_2 + x_3 = 52.5$$

c) State the convergence condition for the Gauss-Seidel method, and interchange rows such that the following system satisfy convergence condition.

$$40x + 5y + 26z = 19$$
$$10x + 7y + 20z = -3$$
$$21x + 30y + 4z = 47$$

Then, perform 3 iterations of Gauss Seidel method to solve the resulting system starting with the initial approximation (0, 0, 0)

Q3. (2X7=14)

a) Using Rayleigh power method, find the largest eigenvalue and corresponding eigenvector of the following matrix with initial approximation $[1,1,1]^T$. Perform only 3 iterations.

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 2 \end{bmatrix}$$

b) Determine the spacing *h* in a table of equally spaced values of the function $f(x) = \sin(x)$ in the interval $\left(0, \frac{\pi}{2}\right)$, so that the interpolation with a second degree polynomial in the table will yield the desired accuracy of six decimal points.

c) The following data set represents the resistivity of a given metal with temperature. Predict the resistivity at 350 K temperature using Strling central difference formula.

| | | | | | |
|---|---|---|---|---|---|
| Temperature (K): | 100 | 200 | 300 | 400 | 500 |
| Resistivity ($\Omega$-cm, $\times 10^6$): | 4.2 | 8.1 | 12.8 | 16.6 | 20.0 |

Q 4. (2X7=14)

**a)** Estimate the integral $I = \int_0^1 \frac{dx}{1+x}$, using Simpson 1/3 formula with eight equal subintervals and compare the result with exact value. Also, Compute the upper bound of the error.

**b)** Find the cubic spline fit for the following data points:

| | | | |
|---|---|---|---|
| $x$: | $-1$ | 0 | 1 |
| $f(x)$: | 2 | 5 | 9 |

Use natural spline conditions $f''(-1) = 0$ and $f''(1) = 0$.

**c)** It is expected from theoretical consideration, that the rate of flow (*F*) is proportional to some power of the pressure (*P*) at the nozzle of a fire hose. Get the least square curve for following experimental data.

| | | | | | |
|---|---|---|---|---|---|
| Flow rate (*F*): | 90 | 110 | 130 | 150 | 170 |
| Pressure (*P*): | 10 | 18 | 28 | 41 | 53. |

Q 5. (2X7=14)

a) Solve the initial value problem $\frac{dy}{dx} = x^2 - \sin(y)$, $y(0) = 1$ to compute $y(0.2)$ by Runge-Kutta 4th order method with step size 0.1.

b) Use Milne predictor-corrector method to compute *y*(0.4) from differential equation $\frac{dy}{dx} = y^2 - x^2$ and following values

| | | | | |
|---|---|---|---|---|
| $x$ : | 0 | 0.1 | 0.2 | 0.3 |
| $y$ : | 1 | 1.11 | 1.25 | 1.42 |

c) Use finite difference approximation to find values of *y*(0.25), *y*(0.5), *y*(0.75) for the BVP

$y'' - y' + y = 0;\ y(0) = 0,\ y(1) = 1$

# CENTRAL UNIVERSITY OF HARYANA

## Second Semester Term End Examinations September 2022

**Programme: GEC** | **Session: 2021-22**

**Semester: Second** | **Max. Time: 3 Hours**

**Course Title: Typesetting in LaTeX** | **Max. Marks: 70**

**Course Code: SBSMAT 01 02 01 GEC 2124**

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**Instructions:**

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5)

a) Give the command in LaTeX to obtain the expression

$$\sin\alpha \pm \sin\beta = 2\sin\tfrac{1}{2}(\alpha \pm \beta)\cos\tfrac{1}{2}(\alpha \mp \beta).$$

b) Write a code in LaTeX to place an equation in a box with a tag outside the box.

c) Explain Inline and Display math environments with examples.

d) Describe the various commands for spacing in math mode in LaTeX.

e) Name the package which is used to put header and footer in a LaTeX document and show its usage.

f) What package(s) do you require to write the multicolumn? Include an example as well.

g) Give the command using PSTricks to draw a circle whose center is located at (a,b) and has radius r.

Q 2. (2X7=14)

a) Write a code in LaTeX for:

- First level item
- First level item
  - Second level item
  - Second level item

* Third level item
* Third level item
  · Fourth level item
  · Fourth level

b) Write a code in LaTeX for

$E = mc^2$

$\mathbf{T} = \frac{V}{v}$

$\mathbf{B} = \mathbf{T} \times \mathbf{N}$

c) Correct the following input as per LaTeX commands: If $$y = α and $ x = β $$ then $ frac{α+1}β+4} =8 $ and in PSTricks, what PS stands for?

Q3. (2X7=14)

a) Write the code in LaTeX to plot the curves $y = \sqrt{x}$ and $y = x^2$ on the same coordinate. Show the square root of a function as a solid curve and the square function as a dashed curve.

b) What is the difference between the following environments in LaTeX?

(i) \ldots and \cdots

(ii) align and align*

(iii) enumerate and itemize.

c) Write a code in LaTeX for typesetting the following expression:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \left[\int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy\right]^{1/2}$$
$$= \left[\int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^2} r\, dr\, d\theta\right]^{1/2}$$
$$= \left[\pi \int_{0}^{\infty} e^{-u} du\right]^{1/2}$$
$$= \sqrt{\pi}$$

Q 4. (2X7=14)

a) Find the errors in the following LaTeX commands, write the corrected version and its output.

```
\Documentclass{article}
\usepackage{amsmath}
\begin{title}
{We have the followings}
\end{title}
\author{XYZ}
\begin{document}
\maketitle
\begin{itemize}
\item $$ x \ge y $
\item $x \le y $
\item x = y
\end{document}
```

b)

i. Write the command in LaTeX to generate the expression

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

ii. What is the output of the command \psarc(1 1) {2} {0} {30} in PStricks.

c)

(i) The symbol Ω can be produced in LaTeX using the command ...............

(ii) .................... command is used to write a binomial coefficient in LaTeX.

(iii) Write the output of the command

$\frac{d} {dx}\left(\int_0^x f(t)\,dt \right) = f(x)$.

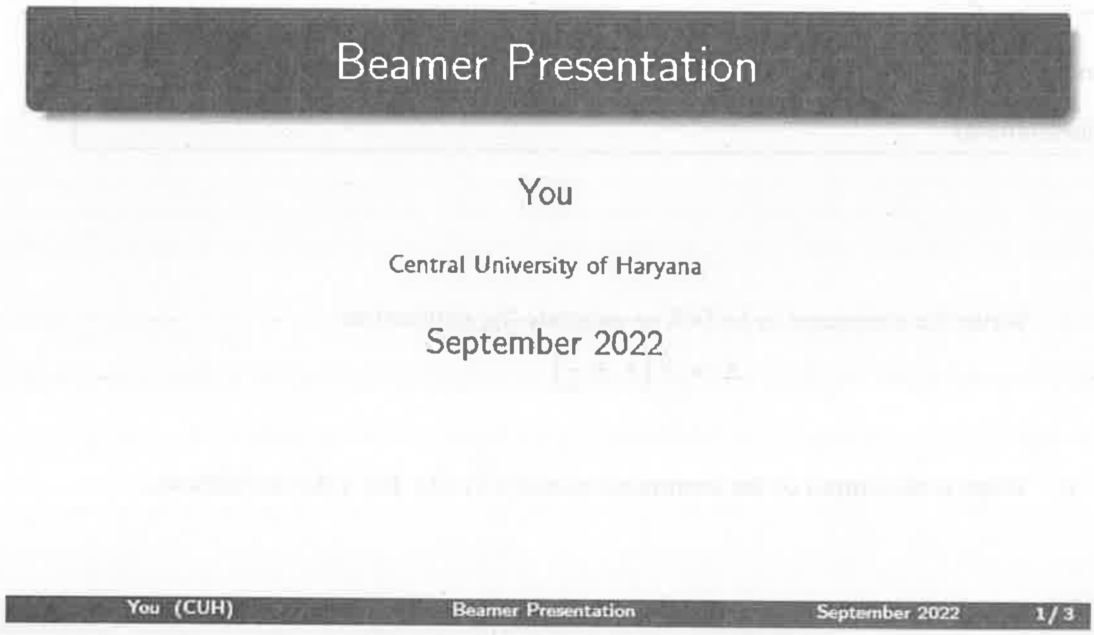
Q 5. (2X7=14)

a) Write a code in LaTeX to typeset the following:

$$\begin{pmatrix} U(t) \\ V(t) \\ W(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos Rt & -\sin Rt \\ 0 & \sin Rt & \cos Rt \end{pmatrix} \begin{pmatrix} U(0) \\ V(0) \\ W(0) \end{pmatrix}.$$

b) How do you insert Ellipses in LaTeX ?

c) Create the following presentation in LaTeX using beamer:



## Introduction

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### Examples

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## Tables

| Item | Quantity |
|---|---|
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Table 1: An example table.

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$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_{i}^{n} X_i$$

denote their mean.