CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: M.Sc. Mathematics | **Session: 2022-23**

Semester: III | **Max. Time: 3 Hours**

Course Title: Integral Equations and Calculus of Variation | **Max. Marks: 70**

Course Code: SBSMAT 01 03 01 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Define the eigenvalues and eigenfunctions of a kernel.

b) Classify the following integral equations $v(x)y(x) = e^x + \lambda \int_0^x xt\, y(t)dt$, $v \neq 0$.

c) For the integral equation $y(x) = 1 + x^3 + \int_0^x 2^{x-t} y(t)dt$, find the iterated kernel $K_3(x,t)$.

d) Solve the following integral equation $y(x) = tanx + \int_{-1}^{1} e^{sin^{-1}x} y(t)dt$.

e) Find the extremals for the functional $J[y] = \int_a^b F(x, y, y')dx$ which has the following Euler's equation $xy'' - y' = 0$.

f) Find the extremals of the functional $J[y] = \int_a^b \frac{\sqrt{1+y'^2}}{y}dx$.

g) Write the convolution theorem for the Laplace transform.

Q 2. (2X7=14)

a) (i) For what value of λ, $y(x) = 1 + \lambda x^2$ is a solution of the following integral equation

$$1 + 2x + x^2 - e^x = \int_0^x e^{x-t} y(t)dt.$$

(ii) Solve $y(x) = 1 + \int_0^x y(t)dt$.

b) Using successive approximations, solve the integral equation

$y(x) = x - \int_0^x (x-t)y(t)dt,\ y_0(x) = 0.$

c) Using resolvent kernel, solve $\int_0^x a^{x-t}\, y(t)dt = f(x),\ f(0) = 0.$

Q3. (2X7=14)

a) (i) Show that the following integral equation $y(x) = \lambda \int_0^1 (t\sqrt{x} - x\sqrt{t})\, y(t)dt$ does not have real eigenvalues and Eigen functions.

(ii) Solve $(x) = x + \lambda \int_0^\pi (1 + sinx\, sint)y(t)dt$.

b) Reduce the following boundary value problem into an integral equation

$y''(x) + \lambda y(x) = x, y(0) = 0, y(\pi) = 0.$

c) Derive the Neumann Series for Fredholm integral equation.

Q 4. (2X7=14)

(a) Show that the Euler's equation of the functional $J[y] = \int_a^b F(x, y, y', y'')dx$ has the first integral $F_{y'} - \frac{d}{dx}F_{y''} = const.$ if the integrand does not depend on y, and the first integral $F - y'\left(F_{y'} - \frac{d}{dx}F_{y''}\right) - y''F_{y''} = const.$ if the integrand does not depend on x.

(b) Find the strong/weak extrema for the functional $J[y] = \int_0^1 \left(\frac{1}{2}y'^2 + yy' + y' + y\right)dx$ with $y(0)$ and $y(1)$ are arbitrary.

(c) Test for an extremum the functional $J[y] = \int_0^a \frac{y}{y'^2}dx$, y(0)=0, y(a)=b, a>0, 0<b<1.

Q 5. (2X7=14)

a) (i) Find the $L^{-1}\left[\frac{s-1}{s^2(s^2+1)}\right]$.

(ii) Find $L\left[\frac{cosat-sinbt}{t}\right]$.

b) (i) Using Laplace transform, solve $\int_0^x \frac{y(t)}{(x-t)^{1/2}}dt = 1 + 2x + x^2$.

(ii) Using Laplace transform, solve $y'(x) = sinx + \int_0^x y(x-t)cost\, dt,\ \ y(0) = 0.$

c) Find the Fourier sine and Fourier cosine transforms of $(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations January 2023

Programme	: Integrated B.Sc.-M.Se. (Mathematics),	Session	: 2022-2023
Semester	: Third	Max. Time	: 3 Hours
Course Title	: Multivariable Calculus	Max. Marks	: 70
Course Code	: SBSMAT 03 03 01 C 5106		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) Find the equation for the tangent plane and normal line at the point P_0 on the given surface $x^2 + 2xy - y^2 + z^2 = 7$, $P_0(1, -1, 3)$.
   (b) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\vec{v} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.
   (c) Evaluate $\nabla.(\vec{r} \times \vec{a})$, where $\vec{a}$ is constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
   (d) Expand $e^x \sin y$ in powers of x and y as far as terms of the second degree.
   (e) If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$.
   (f) Show that $\int_C \phi \nabla \phi . d\vec{r} = 0$, C being a closed curve.
   (g) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate the same.

2. (a) Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as a function of u and v by using the Chain Rule, then evaluate them at the given point (u, v), where $z = 4e^x \ln y$, $x = \ln(u \cos v)$, $y = u \sin v$ and $(u, v) = (2, \frac{\pi}{4})$.
   (b) Let
   $$f(x,y) = \begin{cases} \frac{1}{4}(x^2 + y^2)\log(x^2 + y^2), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}.$$
   Show that $f_{xy} = f_{yx}$ at all points (x, y). Also, show that f_{xy} and f_{yx} are not continuous.
   (c) The surfaces $f(x, y, z) = x^2 + y^2 - 2 = 0$ (A cylinder) and $g(x, y, z) = x + z - 4 = 0$ (A plane) meet in an ellipse E. Find parametric equation for the plane tangent to E at the point $P_0(1, 1, 3)$.

3. (a) State and prove Taylor's Theorem for functions of two variables.
   (b) Verify Euler's theorem for $\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$.
   (c) If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$. Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.

4. (a) Examine the function $x^3 + y^3 - 3axy$ for maxima and minima.
   (b) Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2+y^2+z^2 = 1$ using Lagrange's method of multipliers.
   (c) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find curl$\{(\vec{a} \times \vec{r})r^n\}$, where $r = |\vec{r}|$.

5. (a) Evaluate $\iint r \sin\theta\, dr d\theta$ over the area of the cardioid $r = a(1 + \cos(\theta))$ above the initial line.
   (b) Find the volume of the region S enclosed by the paraboloid $z = 6 - x^2 - y^2$ and $z = 5x^2 + 5y^2$.

(c) Using Spherical coordinate system, evaluate

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} z^2 \sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

6. (a) State and prove Gauss divergence theorem.

(b) Verify Stoke's theorem for $\vec{f} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ when S is the upper half of the surface of sphere $x^2+y^2+z^2=1$ and C is its boundary.

(c) State and prove Fundamental theorem for line integrals.

CENTRAL UNIVERSITY OF HARYANA

Third Semester Term End Examinations January 2023

Programme: M.Sc. **Session: 2022-23**

Semester: Third **Max. Time: 3 Hours**

Course Title: Algebra-II **Max. Marks: 70**

Course Code: SBSMAT 01 03 03 DCEC 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Prove that every abelian group G is a module over the ring of integers Z.

b) Prove that $\sin m^{\circ}$ is an algebraic integer for every integer m.

c) Find the splitting field and its degree for the polynomial x^3-2 over $\boldsymbol{Q}$.

d) If P is the prime subfield of K then prove that $AutK = G(K,P)$.

e) Find rational canonical form of the following matrix over $\boldsymbol{Q}$.

$$\begin{bmatrix} -3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

f) Let A and B be R-submodules of R-modules M and N, respectively. Then prove that

$$\frac{M\times N}{A\times B} \cong \frac{M}{A}\times\frac{N}{B}$$

g) Find the Galois group of splitting field of x^4+1 over $\boldsymbol{Q}$.

Q 2. (2X7=14)

a) Prove that every finite extension is an algebraic extension but converse is not true.

b) Prove that upto isomorphism there are only two prime fields $\boldsymbol{Q}$ and $Z/_{<p>}$.

c) Prove that if characteristic $F=0$ and a, b be algebraic element of F then $F(a,b)$ is simple extension.

Q3. (2X7=14)

a) State and Prove Artin's Theorem.

b) Prove that set of all automorphisms of K form a group under composition of mappings. Also, prove that $G(K,P)$ is a subgroup of $AutK$.

c) State and prove Fundamental theorem of Galois Theory.

Q 4. (2X7=14)

a) State and prove fundamental theorem of finitely generated modules over Euclidean rings.

b) (i) Let R be a ring with unity. Show that an R-module M is cyclic if and only if $M \cong R/I$ for some ideal I of R.

(ii) Let $f: M \to N$ be a homomorphism of an R-module M into an R-module N, then show that the range of homomorphism is an R-submodule of N.

c) Find invariant factors, elementary divisors and Jordon canonical form of the following matrix.

$$\begin{bmatrix} 5 & 1/2 & -2 & 4 \\ 0 & 5 & 4 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Q 5. (2X7=14)

a) State and prove Schur's Lemma.

b) Let M be a free R-module with a basis $\{e_1, e_2, ..., e_n\}$. Then prove that $M \cong R^n$.

c) State and prove Wedderburn-Artin Theorem.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations January-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: Third	Max. Time	: 3 Hours
Course Title	: Functional Analysis	Max. Marks	: 70
Course Code	: SBSMAT 01 03 02 C 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Let (X, d) be any metric space. Show that the function d_1 defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ $\forall x, y \in X$ is a metric on X.
   (b) Show that every convergent sequence is a Cauchy sequence.
   (c) State and prove parallelogram law in Hilbert space.
   (d) Give an example of Banach space which is not a Hilbert space.
   (e) Show that an operator T on Hilbert space H is unitary if and only if it is an isometric isomorphism of H onto itself.
   (f) Define self adjoint operator and gives its example.
   (g) Let N and N^1 be two normed linear space and D is a subspace of N. A linear transformation $T : D \to N^1$ is closed if and only if its graph is closed.

2. (a) State and prove Banach fixed point principle.
   (b) Let p be a real number such that $1 \leq p < \infty$, we denote it by l_p^n the space of all n-tuples of scalars with the norm defined by $||x||_p = (\sum_{i=1}^n |x_i|^p)^{\frac{1}{p}}$ then prove that l_p^n is a Banach space.
   (c) Prove that L_p is a Banach space where $1 \leq p < \infty$.

3. (a) Show that l_p is reflexive space.
   (b) Let N and N^1 be two normed linear space over the same field. Prove that a linear transformation N to N^1 is bounded if and only if it is continuous.
   (c) State and prove Bessel's inequality.

4. (a) Let H be a Hilbert space and let f be an arbitrary functional in H^* then show that there exist a unique vector $y \in H$ such that $f(x) = \ < x, y >$ for every x in H.
   (b) If A is a positive operator on a Hilbert space H then show that $(I + A)$ is non-singular where I is an identity operator.
   (c) Define normal operator and prove that $||N^2|| = ||N||^2$ where N is normal operator on Hilbert space.

5. (a) State and prove uniform boundedness principle.
   (b) State and prove closed graph theorem.
   (c) State and prove Hahn-Banach theorem.

CENTRAL UNIVERSITY OF HARYANA
Third Semester Examinations Jan-2023

Programme	: Integrated B.Sc.-M.Sc. (Mathematics)	Session	: 2022-2023
Semester	: III	Max. Time	: 3 Hours
Course Title	: Group Theory	Maximum Marks	: 70
Course Code	: SBSMAT 03 03 02 C 5106		

Instructions:
1. Question no. 1 has has seven sub parts and students need to answer any five. Each sub part carries two marks.
2. Question no. 2 to 6 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries six marks.

1. (a) Define group. Give an example of non-abelain group with eleboration.
   (b) Show that in a group there is only one identity.
   (c) Define order of an element. Consider $U(15)$ under multiplication modulo 15, find the order of 7, 11 and 13.
   (d) Show that any two disjoint permutations commute.
   (e) Show that center of a group G is a subgroup of G.
   (f) How $U(8)$ is not a cyclic group.
   (g) Define Ring. Give an example of commutative ring with unity and mention the unity.

2. (a) Show that a finite semi-group in which cancellation laws hold is a group.
   (b) Show that symmetries of a square forms a group. Is it commutative.?
   (c) Define quaternion. Show that quaternions form a group.

3. (a) A non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, whenever $a, b \in H$.
   (b) Define Euler phi function. State and prove the Euler's theorem.
   (c) State and prove Lagrange's theorem. Is the converse of Lagrange's theorem true justify your statement.

4. (a) A subgroup H of a group G is normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G.
   (b) Let G be a finite group and suppose p is a prime such that $\frac{p}{o(G)}$, then there exits $x \in G$ such that $o(x) = p$.
   (c) Show that $N(x^{-1}ax) = x^{-1}N(a)x$, for all $a, x \in G$.

5. (a) Define permutation. Show that an odd permutation is of even order.
   (b) Show that every permutation can be written as a product of disjoint cycles.
   (c) Show that every group is isomorphic to a permutation group.

6. (a) Define isomorphism. If $f : G \longrightarrow G'$ is a homomorphism then show that $f(e) = e'$ and $f(x^{-1}) = (f(x))^{-1}$.
   (b) Show that every homomorphic image of a group G is isomorphic to a quotient group of G.
   (c) State and prove the third theorem of isomorphism.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: Integ. BSc-MSc Mathematics | **Session: 2022-23**

Semester: III | **Max. Time: 3 Hour**

Course Title: Computer Fundamentals and Programming in C | **Max. Marks: 70**

Course Code: SBSMAT 03 03 02 SEC 3024

Instructions:

1. *Question No. 1 has seven parts and students are required to answer any five. Each part carries two marks.*
2. *Question No. 2 to 6 have three parts each and students are required to answer any two parts of each question. Each part carries six marks.*

Q 1(a). Which of the following are invalid constants and why?

25,000   3.5e-5   1.5e+2.5 $255

Q 1(b). Write down two differences between structure and union.

Q 1(c). Which of the following arithmetic expressions are valid? If valid, give the value of the expression; otherwise give reason.

15.25 + – 5.0   (5/3)*3 + 5 % 3

Q 1(d). State whether the following statements are true or false.

(i) The purpose of the header file ***<stdio.h>*** is to store the programs created by the users.

(ii) The C standard function that receives a single character from the keyboard is **getchar**.

(iii) The **getchar** cannot be used to read a line of text from the keyboard.

(iv) The input list in a **scanf** statement can contain one or more variables.

Q 1(e). State whether the following are true or false:

(i) When ***if*** statements are nested, the last ***else*** gets associated with the nearest ***if*** without an ***else***.

(ii) One ***if*** can have more than one ***else*** clause.

(iii) A ***switch*** statement can always be replaced by a series of ***if..else*** statements.

(iv) A program stops its execution when a ***break*** statement is encountered.

Q 1(f). Which of the following is the correct syntax of **for** loop?

(i) for (a=0: a<b: a++)

(ii) for (a=0, a<b, a++)

(iii) for (a=0; a<b; a++)

(iv) for (a=0; a++; a<b)

Q 1(g). Fill in the blanks in the following statements.

(i) The parameters used in a function call are called________.

(ii) In prototype declaration, specifying ______ is optional.

(iii) In passing by pointers, the variables of the formal parameters must be prefixed with ____ in their declaration.

(iv) A function that calls itself is known as a ______function.

Q 2(a). Identify syntax errors in the following program. After corrections, what output would you expect when you execute it?

```
#define PI 3.14159
main()
{
int R,C; / * R-Radius of circle
float perimeter; / * Circumference of circle */
float area; /* Area of circle */
C = PI
R = 5;
Perimeter = 2.0 * C *R;
Area = C*R*R;
printf("%f", "%d", &perimeter, &area) }
```

Q 2(b). Discuss the main features of the following looping structures:

for **while** **do..while**

Q 2(c). The numbers in the sequence

1 1 2 3 5 8 13 21.....

are called Fibonacci numbers. Write a program using a looping structure to calculate and print the first m Fibonacci numbers.

Q 3(a). Write a program using one-dimensional array to evaluate the following expression:

$Total = \sum_{i=1}^{10} x_i^2$. The values of x_1, x_2,....are read from the terminal.

Q 3(b). Write a program to find transpose of a 3x3 matrix.

Q 3(c). Fill in the blanks in the following statements.

(i) The variable used as a subscript in an array is popularly known as ______variable.

(ii) An array can be initialized either at compile time or at ________.

(iii) An array created using **malloc** function at run time is referred to as _______array.

(iv) An array that uses more than two subscript is referred to as __________array.

(v) _________is the process of arranging the elements of an array in order.

(vi) In C, by default, the first subscript is ______.

Q 4(a). Explain the following in terms of their scope, visibility and lifetime:

Automatic variables Global variables Static variables

Q4 (b). Write a C program that reads a string and prints if it is a palindrome or not.

Q 4 (c). Describe the main string functions provided in the C language.

Q 5(a). Explain how complex numbers can be represented using structures. Write two C functions: one to return the sum of complex numbers passed as parameters, and the other to return the product of two complex numbers.

Q 5(b). What is recursion. Using this concept, write a function to evaluate factorial of an integer n.

Q5 (c). Write a program to calculate the standard deviation of an array of values. The array elements are read from the terminal. Use functions to calculate standard deviation and mean.

Q 6(a). Describe the process of opening and closing of a file in C, along with the relevant I/O functions and modes. .

Q 6(b). Describe the role of *include* facility in C programming. Illustrate with some suitable examples.

Q 6 (c). Write a program using pointers to compute the sum of all elements stored in an array.

CENTRAL UNIVERSITY OF HARYANA

Third Semester Term End Examinations January 2023

Programme: LL.B. **Session: 2022-23**
Semester: Third **Max. Time: 3 Hours**
Course Title: Gender Justice & Feminist Jurisprudence **Max. Marks: 70**
Course Code: SL LAW 03 03 05 E 4004

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student are required to answer any two parts of each question. Each part carries seven marks.

Q 1. Write short notes on the following: (4X3.5=14)

a) Human Rights of Women under the Universal Declaration of Human Rights, 1948
b) Notions of sex and gender
c) Gender justice and the Directive Principles of State Policy
d) Dowry prohibition officers under the Dowry Prohibition Act, 1961
e) Appropriate Authority and Advisory Committee under the PNDT Act, 1994
f) *Centre for Enquiry into Health and Allied Themes (CEHAT) and others* v. *Union of India and others*, (2001) 5 SCC 2007
g) *Air India* v. *Nargesh Mirza*, AIR 1981 SC 1929

Q 2. (2X7=14)

a) *"One is not born, but rather becomes, a woman. Femininity does not arise from differences in biology, psychology, or intellect. Rather, femininity is a construction of civilization. Biology does not determine what makes a woman a woman—a woman learns her role from man and others in society. Woman is not born passive, secondary, and nonessential, but all the forces in the external world have conspired to make her so. Every individual self, regardless of gender, is entitled to subjectivity; it is only outside forces that have conspired to rob woman of this right."* Do you agree with this statement of *Simone de Beauvoir*? Support your answer with relevant illustrations.
b) The Convention on Elimination of All Forms of Discrimination against Women (CEDAW), 1979 is founded on three basic principles of Substantive Equality, State of Obligation and Non-Discrimination. Discuss with the help of relevant examples.
c) Explain the key features of the Declaration on Elimination of Violence against Women, 1993.

Q3. (2X7=14)

a) "The Constitution of India embodies various provisions relating to gender equality and gender justice." How far these constitutional safeguards been successful in achieving the desired goals? Comment.

b) Discuss the composition, powers and functions of the National Commission for Women (NCW).

c) *Vishaka's* landmark judgement of 1997 finally culminated into the Sexual Harassment of Women at Workplace (Prevention, Prohibition and Redressal) Act, 2013. Do you think that these efforts have led to the women's emancipation and upholding the dignity of women.

Q 4. (2X7=14)

a) Mahatma Gandhi once said *"any young man who makes dowry a condition of marriage discredits his education and his country and dishonors womanhood....A strong public opinion should be created in condemnation of the degrading practice of dowry and young men who soil their fingers with such ill-gotten gold should be excommunicated from society."* Referring to this statement, critically analyze whether any substantial transformation has been brought by the Dowry Prohibition Act, 1961.

b) What do you understand by 'domestic violence.' Explain various types of relief orders which can be passed in favour of the aggrieved person under the Protection of Women from Domestic Violence Act, 2005.

c) Discuss the powers and duties of Protection Officers under the Domestic Violence Act, 2005.

Q 5. (2X7=14)

a) Explain the provisions relating to 'Offences and Penalties' under the Pre-conception and Pre-natal Diagnostic Techniques (Prohibition of Sex Selection) Act, 1994.

b) Millions of our fellow human beings continue to live as contemporary slaves, victims of abominable practices like human trafficking, forced labour and sexual exploitation. Countless children are forced to become soldiers, work in sweat shops or are sold by desperate families. Women are brutalized and traded like commodities. Do you agree that the Immoral Traffic (Prevention) Act 1956 has illegalized human trafficking but it has not been successful in abolishing the same?

c) Discuss the constitution and functions of the Central Supervisory Board under the Pre-conception and Pre-natal Diagnostic Techniques (Prohibition of Sex Selection) Act, 1994.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations January-2023

Programme	: M.Sc. Mathematics	Session	: 2022-23
Semester	: Third	Max. Time	: 3 Hours
Course Title	: Applied Discrete Mathematics	Max. Marks	: 70
Course Code	: SBSMAT 01 03 01 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) What do you mean by Tautology and Fallacy statement?
   (b) Explain disjunction, conjunction, negation and implication. What are their truth tables?
   (c) Define Lattice as a algebraic system.
   (d) Let $A = \{1, 2, 3\}$ and $\subseteq$ be the inclusion relation on $P(A)$. Draw the Hasse diagram of $(P(A), \subseteq)$.
   (e) Discuss AND, OR and NOT gates.
   (f) Define spanning tree.
   (g) Define complete bipartite graph $K_{n,m}$.

2. (a) Show that $\sim (p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.
   (b) Discuss the rules of inference of logics. Show that the following argument is valid.

$$\begin{array}{c} p \\ p \rightarrow q \\ q \rightarrow r \\ \hline r \end{array}$$

   (c) Show that the sets $\{\downarrow\}$ and $\{\uparrow\}$ are functionally complete.

3. (a) Show that dual of complemented lattice is complemented.
   (b) Show that both the definitions of lattice as a partially ordered set and as a algebraic system are equivalent.
   (c) Show that a modular lattice is non-distributive if and only if it contains a sublattice isomorphic with pentagonal lattice M_5.

4. (a) Show that every Boolean Algebra is a lattice.
   (b) Describe the Karnaugh maps for three and four variables. Use a Karnaugh map to find a minimal form of the function $f(x, y, z, w) = xyzw + xyzw' + xy'zw' + x'y'zw + x'y'zw'$.
   (c) State and prove representation theorem for finite Boolean algebras.

5. (a) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
   (b) Show that a tree with n vertices has $n - 1$ edges.
   (c) Derive Euler formula for connected planar graphs.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: B.Sc. - M.Sc. (Integrated) **Session: 2022-23**

Semester: Third **Max. Time: 3 Hours**

Course Title: Introductory Calculus and Analysis **Max. Marks: 70**

Course Code: SBSMAT 03 03 01 GE 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two Marks.

2. Question no. 2 to 6 have three parts and student are required to answer any two parts of each question. Each part carries six marks.

Q 1. (5X2=10)

a) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2y_2 + xy_1 + y = 0$.

b) Evaluate $\int_0^\infty e^{-x^3}\sqrt{x}\,dx$.

c) Discuss the convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$.

d) If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$.

e) If $x = u(1+v), y = v(1+u)$, show that $\frac{\partial(x,y)}{\partial(u,v)} = 1 + u + v$.

f) Determine the constant a so that the vector $\vec{f} = (x+3y)\hat{\imath} + (y-2z)\hat{\jmath} + (x+az)\hat{k}$ is solenoidal.

g) Discuss the convergence of $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^{3/2}}\,dx$.

Q 2. (2X6=12)

a) If $y = (\sin^{-1}x)^2$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. Deduce that $\lim_{x\to 0} y_{n+2} = n^2 y_n$ and find $y_n(0)$.

b) (i) Let $f(x) = \begin{cases} 1, & x \le 3 \\ ax+b, & 3 < x < 5 \\ 7, & x \ge 5 \end{cases}$. Determine the constant a and b so that f is continuous for all x.

(ii) If $f(x) = x^3 + 2x^2 - 5x + 11$, find the value of $f\left(\frac{9}{10}\right)$ with the help of Taylor's series.

c) State and prove Lagrange's Mean Value Theorem.

Q3. (2X6=12)

a) Discuss the convergence of the Beta function.

b) Find the area of the region bounded by $y^2 = 25x$ and $x^2 = 16y$ using double integral.

c) State and prove fundamental theorem of integral calculus.

Q 4. (2X6=12)

a) By definition, show that $\lim_{x\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

b) (i) Examine the convergence of the series: $\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \cdots$

(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n+1}}$.

c) Test the convergence of the series:

$$x^2 + \frac{2^2}{3.4}x^4 + \frac{2^2.4^2}{3.4.5.6}x^6 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8 + \ldots \quad (x > 0)$$

Q 5. (2X6=12)

a) If $z = 2u^2 - v^2 + 3w^2$, where $u = xe^y,\ v = ye^{-x},\ w = \frac{y}{x}$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

b) (i) Find the directional derivative of $\varphi = xy + yz + zx$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ at (1,2,0).

(ii) If $r = |\vec{r}|$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$; prove that $\nabla log|\vec{r}| = \frac{\vec{r}}{r^2}$.

c) Examine the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ for maxima and minima.

Q 6. (2X6=12)

a) Verify Stoke's theorem for $\vec{f} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper half of the surface of sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

b) If $\vec{f} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$; evaluate $\iint_S \vec{f}.\hat{n}\, ds$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

c) Prove that $\nabla^2(r\vec{r}) = \frac{4}{3}\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: M. Sc. (Mathematics) | **Session: 2022-23**

Semester: Third | **Max. Time: 3 Hours**

Course Title: Mathematical Statistics | **Max. Marks: 70**

Course Code: SBSMAT 01 03 03 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half marks.

2. Question no. 2 to 5 have three questions and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Show that the algebraic sum of the deviations of a set of values from their arithmetic mean is zero.

b) Find the constant k such that the function $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$ is a density function and find P(1<x<2).

c) If X and Y are continuous random variables, then show that E(X+Y) =E(X)+E(Y).

d) Find the moment-generating function of a random variable X having the p.d.f.

$$f(x) = \begin{cases} \frac{1}{3} & -1 < x < 2 \\ 0 & otherwise \end{cases}.$$

e) If X is a uniform random variable in [-1, 1], then find mean and variance of X.

f) State Central Limit Theorem.

g) Discuss in brief about type I and type II errors.

Q 2. (2X7=14)

a) Calculate (i) Quartile Deviation (Q.D.) and (ii) Mean Deviation (M.D.) from the mean for the following data:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Students	6	5	8	15	7	6	3

b) Find the coefficient of correlation from the following data by the assumed mean method:

X	10	12	18	16	15	19	18	17
Y	30	35	45	44	42	48	47	46

c) State and prove Baye's theorem.

Q3. (2X7=14)

a) In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target.

b) Let X and Y be two discrete random variables with joint mass function defined by $f_{X,Y}(x,y) = \begin{cases} \frac{1}{4} & (x,y) \in \{(0,0),(1,1),(1,-1),(2,0)\} \\ 0 & otherwise \end{cases}$. Find expectation, variance and covariance.

c) Derive expressions for the mean and variance of geometric distribution.

Q 4. (2X7=14)

a) On the average, a certain computer part lasts 10 years. The length of time the computer part lasts is exponentially distributed. Then
   (i) What is the probability that a computer part lasts more than 7 years?
   (ii) What is the probability that a computer part lasts between 9 and 11 years?

b) Derive expressions for the mean and variance of gamma distribution.

c) Show that the mean deviation from the mean of the normal distribution is about 4/5 of its standard deviation.

Q 5. (2X7=14)

a) A sales clerk in the departmental store claims that 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed that 35 of them left without buying anything. Are these sample results consistent with the claim of the sale clerk? Use a 5% level of significance.

b) Two independent samples of 8 and 7 items gave the following values:

Sample A	9	11	13	11	15	9	12	14
Sample B	10	12	10	14	9	8	10	

Examine whether the difference between the means of two samples is significant at 5% l.o.s. (The value of t for 13 d.f. at 5% l.o.s. is 2.16).

c) A sample analysis of examination results of 200 MSc's was made. It was found that 46 students had failed, 68 secured a third position, 62 secured a second division and the rest were placed in first position. Are these figures commensurate with the general examination result which is in the ratio of 4:3:2:1 for various categories respectively? (The table value of chi-square for 3 d.f. at 5% l.o.s. is 7.815)

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations January 2023

Programme: Integrated B.Sc.-M. Sc. (Mathematics) **Session: 2022-23**

Semester: Third **Max. Time: 3 Hours**

Course Title: Probability & Statistics **Max. Marks: 70**

Course Code: SBSMAT 03 03 03 C 5106

Instructions:

1. Question no. 1 has seven parts and students are required to answer any five. Each part carries two marks.

2. Questions no. 2 to 6 have three questions and students are required to answer any two parts of each question. Each part carries six marks.

Q 1. (5X2=10)

a) Let A and B be two events such that P(A) =0.3 and $P(A \cup B)$=0.8. If A and B are independent events than P(B) = ...

b) Write the probability axioms.

c) The variance of the binomial distribution $\binom{10}{x}\left(\frac{2}{3}\right)^x\left(\frac{3}{5}\right)^{10-x}$ $; x = 0,1,2 \ldots,10$ is .....

d) Define moment-generating function.

e) Define weak law of large numbers (WLLN).

f) The joint pdf of random variables in bivariate normal distribution is ....

g) A coin is biased so that the probability of head is 0.60. The entropy is ......

Q 2. (2X6=12)

a) State and prove Baye's theorem.

b) The diameter of an electric cable, say X, is assumed to be a continuous random variable with : $f(x) = 6x(1-x),\ 0 \leq x \leq 1$.
(i) Check that f(x) is p.d.f. and
(ii) Determine a number b such that $P(X < b) = P(X > b)$.

c) What do you mean by moment generating function? A coin is tossed until a head appears. What is the expectation of the number of tosses required?

Q3. (2X6=12)

a) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain: (i) no defective, and (ii) at least two defectives. [Given $e^{-0.5} = 0.6065$]

b) Show that the mean deviation from the mean of the normal distribution is about 4/5 of its standard deviation.

c) Derive expressions for the mean and variance of Chi-Square distribution.

Q 4. (2X6=12)

a) A two-dimensional random variable (X, Y) have a bivariate distribution given by: $P(X = x, Y = y) = \frac{x^2+y}{32}, for\ x = 0, 1, 2, 3\ and\ y = 0, 1.$ Find the marginal distributions of X and Y.

b) The joint probability density function of a two-dimensional random variable (X, Y) is given by: $f(x, y) = \begin{cases} 2; & 0 < x < 1, 0 < y < x \\ 0 & elsewhere \end{cases}$

(i) Find the conditional density function of Y given X = x and conditional density function of X given Y = y.

(ii) Check for independence of X and Y.

c) If X and Y are continuous random variables, then show that E(X+Y) =E(X)+E(Y).

Q 5. (2X5=10)

a) Calculate coefficient of rank correlation from the following data:

X	15	10	20	28	12	10	16	18
Y	16	14	12	12	11	15	18	12

b) Define covariance of random variables. State and prove at least three properties of covariance.

c) State and prove Chebyshev's theorem.

Q 6. (2X6=12)

a) What do you mean by information? Discuss in detail about component of the information.

b) Show that H (X, Y) = H(X) + H(Y/X).

c) Discuss in detail about Polya's urn model.

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations January 2023

Programme: M. Sc. (Mathematics) **Session: 2022-23**

Semester: Second **Max. Time: 3 Hours**

Course Title: Linear Algebra **Max. Marks: 70**

Course Code: SBSMAT 01 02 01 C 3104

Instructions:

1. Question no. 1 has seven parts and students are required to answer any four. Each part carries three and half Marks.

2. Questions no. 2 to 5 have three questions and students are required to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) Show that two vectors in a vector space are linearly dependent if and only if one of the vectors is a multiple of the other.

b) State Extension and Existence theorems.

c) Show that similar matrices have the same characteristic polynomials.

d) What do you think about the change of basis in linear transformation? Illustrate it with an example.

e) State Primary Decomposition theorem.

f) Define Invariant subspace. Show that kernel of a linear operator is invariant.

g) Find the angle between $f(t) = t - 1$ *and* $g(t) = t$ in the polynomial space $P(t)$ with inner product$< f, g >= \int_0^1 f(t)g(t)dt$.

Q 2. (2X7=14)

a) Prove that the union of two subspaces of a vector space is a subspace of the vector space if and only if one is contained in the other.

b) Let W be a subspace of a finite-dimensional vector space V. Then prove that $\dim \frac{V}{W} = \dim V - \dim W$.

c) Find range, rank, kernel and nullity of the linear transformation defined by $T: \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^2$ such that $T(x, y, z) = (x + y, 2z - x)$.

Q3. (2X7=14)

a) Let S, T be two linear transformations from V(F) into V(F). Let B be an ordered basis of V. Then prove that $[ST]_B = [S]_B[T]_B$.

b) State and prove the Cayley-Hamilton theorem.

c) Let T be a linear operator on $\boldsymbol{R}^3(\boldsymbol{R})$ defined by $T(x, y, z) = (3x + y - z, 2x + 4y - 2z, -x - y + 3z)$. Find the characteristic polynomial and minimal polynomial for T.

Q 4. (2X7=14)

a) Determine all the possible Jordan canonical forms for a linear operator $T: V \rightarrow V$ whose characteristic polynomial is $(t-2)^3(t-5)^2$.

b) Let f be a bilinear form on a vector space V(F) and the characteristic of F is different from 2. Then, prove that f is skew-symmetric if and only if f is alternating bilinear form.

c) Find the nature, index, signature, and rank of the following quadratic form without reducing to canonical form: $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$.

Q 5. (2X7=14)

a) State and prove Cauchy-Schwarz's inequality. Also, discuss the case when equality holds.

b) Let V be the inner product space of all polynomials of degree 2 or less in indeterminate t with inner product defined by $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt$.

Apply Gram-Schmidt orthogonalization process to obtain an orthonormal basis from the basis $\{1, t, t^2\}$.

c) A necessary and sufficient condition that a self-adjoint operator T on a finite dimensional inner product space V be 0 (null operator) is that $\langle T(u), u\rangle = 0 \; \forall \; u \; in \; V$.