CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations May/June.-2022

Programme	: M.Sc. Mathematics (Re-appear)	Session	: 2021-22
Semester	: Second	Max. Time	: 3 Hours
Course Title	: Topology	Max. Marks	: 70
Course Code	: SPMMAT 01 02 02 C 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define closed set and if F is closed set, then F^c, that is $X - F$ is open.

   (b) Prove that
   i) $i(A) = A - b(A)$
   ii) $C(A) = A \cup b(A)$
   where $i(A)$, $C(A)$ and $b(A)$ denote interior of A, closure of A and boundary of A respectively.

   (c) Define projection mapping in product space.

   (d) State Pasting lemma.

   (e) Give an example of a space which is locally connected but not connected.

   (f) Give an example of a space which is T_0 but not T_1.

   (g) Define normal space, regular space and state Tietz extension theorem.

2. (a) Let X be an infinite set and T be the family consisting of ϕ and complements of finite subset of X. Show that T is a topology on X.

   (b) Let (X, T) be a topological space . A family $\mathbb{B}$ of T is a base for T if and only if every open set can be expressed as union of member of $\mathbb{B}$.

   (c) If A, B and E are subset of the topological space (X, T), then the derived set has following properties
   i) $d(\phi) = \phi$
   ii) If $A \subseteq B$ then $d(A) \subseteq d(B)$
   iii) If $x \in d(E)$, then $x \in d(E - \{x\})$
   iv) $d(A \cup B) = d(A) \cup d(B)$.

3. (a) Let (X, T) be a topological space and A be a subset of X. Let $\mathbb{B}$ be a basis for T. Then $B_A = \{B \cap A : B \in \mathbb{B}\}$ is a basis for the subspace topology on A.

   (b) Let (X, T) and (X^*, T^*) be two topological spaces. A one to one mapping f from X onto X^* is homeomorphism if $f(C(E)) = C(F(E))$ for every $E \subseteq X$ and $C(E)$ denotes closure of E.

   (c) State and prove Tychnoff theorem.

4. (a) Prove that a subspace of real line $\mathbb{R}$ is connected if and only if it is an interval. In particular $\mathbb{R}$ is connected.

   (b) A topological space (X, T) is compact if and only if any family of closed sets having the finite intersection properties has a non empty intersection.

(c) Let (X, T) be a compact locally connected space. Then (X, T) has a finite number of components.

5. (a) Show that every compact Hausdorff space is normal space.

   (b) Prove that in a second axioms space, every open covering of a subset is reducible to a countable sub-covering that is every second axioms space is Lindeloff space.

   (c) State and prove Urysohn's Lemma.

CENTRAL UNIVERSITY OF HARYANA
End Semester Examinations May/June.-2022

Programme	: M.Sc. Mathematics	Session	: 2021-22
Semester	: Fourth	Max. Time	: 3 Hours
Course Title	: Measure Theory and Integration	Max. Marks	: 70
Course Code	: SBSMAT 01 04 08 DCEC 3104		

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Define Borel set and gives its example. $(4 \times 3.5 = 14)$
   (b) Prove that [0, 1] is uncountable set.
   (c) If f is measurable function defined on E then the set $\{x \in E : f(x) = \alpha\}$ is measurable for each extended real number α.
   (d) Show that the function defined on a set of measure zero is measurable.
   (e) Give an example of Lebesgue integral function that is not Riemann integral.
   (f) Let f and g be bounded measurable functions defined on a set E of finite measure then $|\int_E f| \leq \int_E |f|$.
   (g) Show that if f and g are measurable functions such that $|f| \leq |g|$ a.e. and g is integrable, then f is integrable.

2. (a) Prove that the outer measure of an interval is its length. $(2 \times 7 = 14)$
   (b) Let $\{E_n\}$ be an increasing sequence of measurable sets and $m(E_1) < \infty$, then $m(\cup_{n=1}^{\infty} E_n) = lim_{n\to\infty} m(E_n)$.
   (c) State and prove Littelwood's first principle.

3. (a) Let f be a function defined on a measurable set E. Then f is measurable if and only if for any open set G in $\mathbb{R}$, the inverse image $f^{-1}(G)$ is a measurable set. $(2 \times 7 = 14)$
   (b) Show that if f is measurable function then $|f|$ is also measurable function. Is the converse true? Justify.
   (c) Define relation between simple and step functions and prove that every simple and step functions are always measurable.

4. (a) Let E be a measurable set with $m(E) < \infty$ and $\{f_n\}$ be a sequence of measurable functions which converges to f a.e. on E. Then given $\eta > 0$, there is a set A with $m(A) < \eta$ such that the sequence $\{f_n\}$ converges to f uniformly on $E - A$. $(2 \times 7 = 14)$
   (b) If a sequence $\{f_n\}$ converges in measure to f on E, then there exist a sub-sequence $\{f_{n_k}\}$ which converges to f a.e. on E.

(c) Let f be a function defined on $[0, \frac{1}{\Pi}]$ as follows

$$f(x) = \begin{cases} 0.1 \ \ if \ \ x = 0 \\ 2xsin\frac{1}{x} \ \ if \ \ x > 0 \end{cases}$$

determine the measure of the set $\{x : f(x) \geq 0\}$.

5. (a) State and prove monotone convergence theorem. $(2 \times 7 = 14)$

(b) Find the Lebesgue integral of $\int_0^1 \frac{xsinx}{1+(nx)^\alpha} dx$, where $\alpha > 1$.

(c) State and prove Fatou's Lemma.

Central University of Haryana
End Semester Examination, June-2022

Programme	: M.Sc. Mathematics	Session	: 2021-2022
Semester	: IV	Max. Time	: 3 Hours
Course Title	: Functional Analysis	Max. Marks	: 70
Course Code	: SBSMAT 01 04 09 DCEC 3104		

Instructions:

1 Question no. 1 has seven sub parts and students need to answer any four sub parts. Each sub part carries three and half marks.

2 Question no. 2 to 5 have three sub parts each and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Q1 a) Define metric space. Is every cauchy sequence convergent Justify. $(4 \times 3.5 = 14)$

b) Write down four main differences between Banach and Hilbert spaces.

c) Define norm linear space. Is every metric space a normed linear space justify your statement.

d) Show that on a finite dimensional vector space, all the norms are equivalent.

e) Give the definition of Hilbert space and an example which is not a Hilbert space.

f) Define sublinear functional. Let $\mathbb{X}$ be a real normed linear space and suppose $f(x_0) = 0; \forall f \in \mathbb{X}^*$. Then show that $x_0 = 0$.

g) Let H be a Hilbert space and $T \in B(H)$. If T is self-adjoint, then show that $\langle Tx, x \rangle$ is real $\forall x \in H$.

Q2 a) Define contraction mapping. State and prove Banach fixed point theorem. $(2 \times 7 = 14)$

b) Define bounded linear functional. Show that $B(\mathbb{X}, \mathbb{C})$ is a Banach space under sup norm.

c) Define First Category. Show that every complete metric space is of second category.

Q3 a) Show that norm $\|\|$ is a continuous function from $\mathbb{X}$ to $\mathbb{R}$. Let $\{e_1, e_2, e_3, ..., e_n\}$ be a finite orthonormal set in an inner product space $\mathbb{X}$. Then for any $x \in \mathbb{X}$ show that

$$\sum_{i=1}^{n} |\langle x, e_i \rangle|^2 \leq \|x\|^2.$$

$(2 \times 7 = 14)$

b) Let $T : D(T) \longrightarrow Y$ be a linear operator, where $D(T) \subset \mathbb{X}$ and $\mathbb{X}$, Y are normed linear spaces. Then show that T is continuous if and only if T is bounded. Also, show that if T is continuous at a single point then it is continuous on $D(T)$.

c) Show that the dual of l^p space is l^q space.

Q4 a) State Riesz Representation theorem. Let H_1 and H_2 be two Hilbert spaces and $T : H_1 \longrightarrow H_2$ be a bounded linear operator. Then there exists a unique bounded linear operator $T^* : H_2 \longrightarrow H_1$ such that $\langle Tx, y \rangle = \langle x, T^*y \rangle$ for all $x \in H_1$ and $y \in H_2$. Moreover, $\|T^*\| \leq \|T\|$.

$(2 \times 7 = 14)$

b) Define normal and self adjoint operators. Show that product of two self-adjoint operators S and T is self adjoint if and only if $ST = TS$. Is every normal operator self adjoint. Justify your statement.

c) Define weak convergence. Let $T : H \longrightarrow H$ be a bounded linear operator on a Hilbert space H. Then the following statements are equivalent:

(i) $TT^* = I$

(ii) $\langle Tx, Ty \rangle = \langle x, y \rangle, \forall x, y \in H$

(iii) $\|Tx\| = \|x\|$.

Q5 a) State and prove Hahn-Banach theorem for real vector spaces. $(2 \times 7 = 14)$

b) State and prove Uniform Boundedness Principle.

c) Define closed linear operator. State and prove closed graph theorem.

CENTRAL UNIVERSITY OF HARYANA

End Semester Examinations June 2022

Programme: M.Sc. Mathematics | **Session:** 2021-22

Semester: 2nd **(Reappear)** | **Max. Time:** 3 Hours

Course Title: Numerical Analysis | **Max. Marks: 70**

Course Code: SPMMAT 01 02 03 C3104

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.

2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Q 1. (4X3.5=14)

a) The true value of e (exponential) correct to 10-significant digits is 2.718281828. Calculate absolute and relative errors, if we approximate this value by 2.718.

b) In the Bisection algorithm, let M denotes the length of the interval [a, b]. Let $\{x_0, x_1, ...\}$ represents successive midpoints generated by the Bisection method. Show that, $|x_{i+1} - x_i| = M/2^{i+2}$.

c) Use the definitions of various finite differences to prove the result $\Delta^3 f_1 = \nabla^3 f_4 = \delta^3 f_{5/2}$.

d) Write the Bessel and Stirling formulas for interpolation.

e) Solve the IVP $\frac{dy}{dx} = -2(x + y^2),\ y(1) = 0$; in the range $1 \le x \le 1.4$ using Euler method. Use step size h = 0.2.

f) State Milne and Adams predictor and corrector formulas for solution of IVP

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0.$$

g) Classify the PDE $\frac{\partial^2 u}{\partial x^2} + (x + y)\frac{\partial^2 u}{\partial y^2} + xu = 0$.

Q 2. (2X7=14)

a) Find the order of convergence of Newton Raphson method.

b) Solve the following system of linear equations by Gauss elimination method with partial pivoting using three significant digits floating points rounding arithmetic.

$$2x_1 + x_2 + 2x_3 = 4$$
$$2x_1 - x_2 + 100x_3 = 100$$
$$3x_1 + 50x_2 + x_3 = 52.5$$

c) Perform four iterations of Gauss-Seidel iterative method to solve the following system of linear equations. Consider the initial approximation is $x_1^{(0)} = 0,\ x_2^{(0)} = 0,\ x_3^{(0)} = 0$.

$$16x_1 + 2x_2 - 3x_3 = 34$$
$$23x_1 + 42x_2 + 4x_3 = 88\ .$$
$$5x_1 - 9x_2 + 32x_3 = 1$$

Q3. (2X7=14)

a) The growth of cell culture (optical density) at various pH levels are tabulated in the following table.

pH:	4	4.5	5	5.5	6
Optical density:	0.28	0.35	0.41	0.46	0.52

Compute the optical density at pH level 5.8.

b) Determine the spacing *h* in a table of equally spaced values of the function $f(x) = \sin(x)$ in the interval $\left(0, \frac{\pi}{2}\right)$, so that the interpolation with a second degree polynomial in the table will yield the desired accuracy of six decimal points.

c) Approximate the values $f'(1.2)$ and $f''(1.2)$ for the following data set.

x:	1	1.25	1.5	1.75	2.0
$f(x)$	0	0.223144	0.405465	0.559616	0.693147

Q 4. (2X7=14)

a) Estimate the integral $I = \int_0^1 \frac{dx}{1+x}$, using Trapezoidal formula with eight equal subintervals and compare the result with exact value. Compute the upper bounds of the error.

b) Solve the initial value problem $\frac{dy}{dx} = y - \frac{1}{x}y^2,\ y(1) = 1$ to find $y(1.1)$ by Runge-Kutta method of order four with step size 0.1.

P.T.O.

c) Use modified Euler method to find an approximate value of variable u when x = 1.2 for the IVP

$$\frac{du}{dx} = x^2 + u; \ u(1) = 2 .$$

Q 5. (2X7=14)

a) Solve the 1 – dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}; \qquad 0 \le x \le 1$$

initial condition $u(x,0) = x^2(2-x)$, and

boundary conditions $u(0,t) = 0$ and $u(1,t) = 1$.

Use Explicit scheme to find the values of $u(x, t)$ up to $t = 0.01$, with $\Delta x = 0.25$ and $\Delta t = 0.005$

b) Prove that the Crank-Nicolson scheme is always convergent for a heat conduction equation of order two in one space variable

c) Solve the Laplace equation $\nabla^2 u = u_{xx} + u_{yy} = 0$ for the square mesh with the boundary values (Dirichlet conditions) as shown in the following figure. Use Gauss elimination method.