CENTRAL UNIVERSITY OF HARYANA

Term End Examination June 2023

Programme:	**M.Sc. Statistics**	**Session:**	**2022-23**
Semester:	**Fourth**	**Max. Time:**	**3 Hours**
Course Title:	**Survival Analysis**	**Max. Marks:**	**70**
Course Code:	**SBS ST 01 402 DCE 3104**		

Instructions:

1. Question number 1 has seven parts and students need to answer any four. Each part carries three and a half Marks.
2. Questions number 2 to 5 have three parts and students need to answer any two parts of each question. Each part carries seven marks.

Question No. 1. **(4x3.5=14)**

a. What do you understand by Survival Analysis? What is the difference between Survival Analysis and Reliability Theory, explain with at least one example.

b. Prove that for a continuous random variable $T > 0$, $MTSF = \int_0^\infty R(t)\,dt$.

c. Define $\eta(t) = f'(t)/f(t)$.

(i) If $\eta(t) \in D$, then prove that the failure rate function is decreasing.

(ii) If $\eta(t) \in UBT$ and if there exists a $y_0 > 0$ such that $g'(y_0) = 0$, then prove that failure rate function is Upside down Bathtub shaped.

d. Discuss the role of censoring in life-testing experiments.

e. Discuss the random censoring with its density function.

f. What do you mean by empirical cumulative distribution function (ECDF)? Obtain the confidence bands for survival function using ECDF.

g. Discuss the Competing risk model.

Question No. 2 **(2x7=14)**

a. Discuss the shape of the failure rate function of the two-parameter gamma distribution.

b. Discuss in detail the properties of IFR and DFR classes.

c. Discuss in detail the Total Time on Test (TTT) Transform. Also, discuss the conditions of TTT transform for the IFR, DFR, BT and UBT classes.

Question No. 3 **(2x7=14)**

a. Obtain the maximum likelihood estimators of the survival function of gamma distribution under Type II censoring.

b. Describe in detail the procedure for obtaining the maximum likelihood estimators of the parameters and survival function of the Weibull distribution using time censored sample.

c. Obtain the maximum likelihood estimator for the failure rate function of the log-logistic lifetime model in case of random censoring having a common shape and different scale parameters.

Question No. 4 **(2x7=14)**

a. Discuss in detail the actuarial estimator for survival function in the censored sample case. Also, obtain the variance of the estimator.

b. Define the Kaplan-Meier estimator and derive the Greenwood formula.

c. Discuss Deshpande's test for exponentiality against increasing failure rate average alternatives.

Question No. 5 **(2x7=14)**

a. What do you mean by two sample non-parametric problems? Describe in detail Gehan's Test for two sample problem.

b. Stating the hypotheses tested, describe the log-rank test. Also, describe the Mantel-Haenzel test.

c. Describe the Cox proportional hazard model and discuss the estimation of the baseline hazard rate function. Based on this how would you estimate the survival function? Deduce the form of the estimator in the absence of covariates.

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Term End Examinations, June 2023

Programme:	**M.Sc. Statistics**	**Session: 2022-2023**
Semester:	**IV**	**Max. Time: 3 Hours**
Course Title:	**Order Statistics**	**Max. Marks: 70**
Course Code:	**SBS ST 01 401 DCE 3104**	

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.

2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Define order statistics with applications.

b) Explain what are the importance of recurrence relation?

c) Define quantile of order p of a continuous distribution.

d) State and prove the probability integral transformation.

e) Explain the concept of record values with example.

f) Explain the concept generalized order statistics.

g) Define the range and find the distribution of the order statistics of $W_{r,s}$ when $f(x)$ is Uniform.

Question No. 2. (2X7=14)

a) Let $X_1, X_2, \ldots X_n$ be a random of size n and $X_{1:n} \le X_{2:n} \le \ldots X_{n:n}$ be the corresponding order statistics from continuous population with probability density function $f(x)$ and cumulative distribution function $F(x)$. Then show that

$$f_{r_1,r_2,\ldots,r_k}(x_1, x_2 \ldots x_k) = \frac{n!}{(r_{i+1} - r_{i-1})!} \prod_{i=1}^{k} f(x_i) \prod_{i=0}^{k} [F(x_{i+1}) - F(x_i)]^{r_{i+1}-r_{i-1}},$$

$-\infty < x_1 < x_2 < \ldots < x_k < \infty$.

b) Let X be a discrete random variable taking the values $x = 0,1,2,\ldots,c$, where c is a positive integer. Obtain the *pmf* of the range_W is sample of size n, when $W > 0$.

c) If $F_{r:n}(x)$ denotes the *cdf* of the rth order statistics in a random sample of size *n*, show that

$$F_{r:n}(x) = [F(x)]^r \sum_{i=1}^{n-r+1} \binom{n-i}{r-1} [1 - F(x)]^{n-r+1-i}.$$

Question No. 3. (2X7=14)

a) If X is continuous with strictly increasing cumulative distribution function $F(x)$, then `show that the random interval $(X_{r:n}, X_{s:n})$, $r < s$, cover ξ_p with a probability which depends only on r, s, n, p but not on $F(x)$ i. e

$P(X_{r:n} \le \xi_p \le X_{sn}) = \pi(r,s,n,p)$ a function of r,s,n,p.

b) Let $Y_{1:n} \le Y_{2:n} \le Y_{3:n}$ be the order statistics of a random sample of size 3 from the uniform $U(0.\theta)$ Distribution. Show that $4Y_1$ and $2Y_2$ are both unbiased estimator for θ and also, find their variances.

c) Prove that the order statistics $X_{1:n} \leq, X_{2:n} \leq, \ldots, \leq X_n$ in a random sample of size n from a population having continuous and strictly increasing cdf $F(x)$ from a markov chain.

Question No. 4. (2X7=14)

a) Let $X_{1:n} \leq, X_{2:n} \leq, \ldots, \leq X_n$ be n order statistics from a continuous population with strictly increasing cumulative distribution function $F(x)$ and let $U = F(x)$ be the probability integral transformation. Show that the variates $V_r = \dfrac{U_{(r)}}{U_{(r+1)}} = \dfrac{F(X_{r:n})}{F(X_{r+1:n})}$, $r = 1,2,\ldots,(n-1)$ are mutually independent. Also show that the variance $V_r^{\,r}$ are mutually independent uniform variate over $U(0,1)$.

b) Let $X_1, X_2, \ldots X_n$ be independent variate, X_i having a geometric distribution with parameter p_i. Show that $X_{1:n}$ is distributed geometrically with parameter $1 - q_1 q_2 \ldots q_n = 1 - \prod_{i=1}^{n} q_i$.

c) If (X,Y) is an observation from a bivariate normal $N(0,0,1,1,\rho)$ population, show that the the expected value of max (X,Y) is $\left[\dfrac{1-\rho}{\pi}\right]^{1/2}$ and variance is $Var[\max(X,Y)] = 1 - \dfrac{1-\rho}{\pi}$.

Question No. 5. (2X7=14)

a) For an arbitrary distribution and $1 \leq r < s \leq n$ then show that

$$(r-1)\mu_{r,s:n} + (s-r)\mu_{r-1,s:n} + (n-s+1)\mu_{r-1,s-1:n} = n\mu_{r-1,s-1:n-1}.$$

b) For a random sample from a standard normal distribution then show that

$$I_{2m+1}(a) = \sum_{i=1}^{2m+1} (-1)^{i+1} \left(\frac{1}{2}\right)^i \binom{2m+1}{i} I_{2m+1-i}(a), \; m = 0,1,2,\ldots$$

c) Show that in random sample from a continuous distribution with cumulative distribution function $F(x)$

$$E[X_{s:n} F(X_{r:n})] = \frac{r}{n+1} \mu_{s+1:n+1}, \; r < s$$

and

$$E[X_{r:n} F(X_{s:n})] = \mu_{r:n} - \frac{n+1-s}{n+1} \mu_{r:n+1}, \; r < s.$$

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Second Semester Term End Examinations July 2023

Programme : M.A/ M.Sc.
Semester : Second
Course Title : Applied Statistics
Course Code : SBS ST 01 201 GE 3104

Session: 2022-23
*Max. Time: **3 Hours***
*Max. Marks: **50***

Instructions:

1. Question no. 1 has five parts and students need to answer any four. Each part carries two and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries five marks.

Note: Scientific Non programmable Calculator is allowed

Question No. 1.
(4x2.5=10)

a. What are chance and assignable causes of variability? What part do they play in the operation and interpretation of a Shewhart control chart?

b. Define statistical quality control.

c. Fill in the blank of the following table which are marked with ?

X	l_x	d_x	q_x	L_x	T_x
20	693435	?	?	?	35081126
21	690673	-	-	-	-

d. Define time series. Explain it with the help of suitable examples.

e. What do you understand by cyclic variation in a time series? Give one example.

Note: Question number **Two to Five** have three sub parts and students need to answer **any two sub part** of each question. Each sub part carries **five marks.**

Question No. 2
(2x5=10)

a. For the following data fit an exponential curve by using least square method

Year	2002	2003	2004	2005
Sale	27.8	62.1	110	161

b. What do you understand by seasonal fluctuation? Explain in detail the semi average method using suitable example.

c. Using Ratio to Trend method to determine the quarterly seasonal indices for the following data.

Year	Quarterly production in (thousands)			
	Quarter I	Quarter II	Quarter III	Quarter IV
2008	800	920	880	880
2009	540	760	680	620
2010	400	580	540	480
2011	340	520	500	440
2012	300	400	360	340

Question No. 3 **(2x5=10)**

a. Explain in detail the criteria for detecting lack of control in mean and range charts.

b. What are control charts for attributes? Derive control limits for controlling the fraction defectives when sample size is not fixed, giving clearly the statistical concept used.

c. What is the statistical justification for using the three sigma limits in the control charts irrespective of the actual probability distribution of the quality characteristic?

Question No. 4 **(2x5=10)**

a. Define vital statistics. What are vital events? Describe the usual sources of data collection on vital events.

b. Describe age specific death rate. Explain how to can find crude death rate from age specific death rate.

c. Compute the standardised death rates by direct and indirect methods for the data given below:

Age Group Years	Population A		Standard Population		
	Population in thousand	Specific death rate	Population in thousand	Specific death rate	
Under 10	14	50	9		52
10 – 20	12	14	12		16
20 – 60	17	8	29		9
≥ 60	12	54	8		55

Question No. 5 **(2x5=10)**

a. Prove the following results

(i) $_{n}q_{x} = \frac{d_{x+n-1}}{l_{x}}$

(ii) $P_{x} = \frac{e_{x}}{1+e_{x+1}}$

b. Distinguish between fertility and fecundity. Explain the crude birth rate and specific fertility in detail.

c. Calculate the General Fertility Rate and Total Fertility Rate from the following data. Assuming that for every 100 girls, 109 boys are born:

Age of Woman	No. of Woman	Age specific fertility rate (per 1000)
15 – 19	212619	98.0
20 – 24	198732	169.6
25 – 29	162800	158.2
30 – 34	145362	139.7
35 – 39	128109	98.6
40 – 44	106211	42.8
45 - 49	86753	16.9

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Term End Examinations, July 2023

Programme	**: M.Sc. Statistics**	
Semester	**: I**	**Session: 2022-2023**
Course Title	**: Distribution Theory**	*Max. Time :* ***3 Hours***
Course Code	**: SBS ST 01 103 C 3104**	*Max. Marks :* ***70***

Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.

Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. **(4X3.5=14)**

i) X has a Poisson distribution with parameter λ. Conditional distribution of Y given x is binomial with parameter (x, p). Find the marginal distribution of Y.

ii) Show that in a Poisson distribution with usual mean, the mean deviation about mean is $2e^{-1}$ times of standard deviation.

iii) The random variable X and Y are independent. X is Poisson (λ) and Y is Cauchy $C(0, \lambda)$. Then find the characteristic function of XY.

iv) 2% of the items made by a machine are defective. Find the probability that 3 or more items are defective in a sample of 100 items.

v) Discuss, in brief, mixture distribution with an example describing its application in real life.

vi) Explain compound and truncated distributions.

vii) Define central and non-central t and χ^2 distributions.

Question No. 2. **(2X7=14)**

i) Two movie theaters complete for the business of 1000 customers. Assume that each customer chooses between the movie theaters independently and with indifference. Let N denote the number of seats in each theater.

a. Using a binomial model, find an expression for N that will guarantee that the probability of turning away a customer (because of a full house) is less than 1%.

b. Use the normal approximation to get a numerical value for N.

ii) Show that under certain conditions, negative binomial distribution tends to Poisson distribution.

iii) For a Binomial distribution, obtain $E\,|\,X - np\,|$.

Question No. 3. **(2X7=14)**

i) In an examination marks obtained by the student in Mathematics, Physics and Statistics are distributed normally about means 50, 52, and 18 with standard deviation is 15, 12,and 16 respectively. Find the probability of securing total marks of (a) 180 or above (b) 90 or below.

ii) Show that if X_1 and X_2 are standard normal variates with correlation coefficient ρ between them, then the correlation coefficient between X_1^2 and X_2^2 is given by ρ^2.

iii) For a Laplace distribution, show that $E(X-\mu)^{2r} = \sigma^{2r}\sqrt{(2r+1)}$.

Question No. 4. **(2X7=14)**

i) Let X_i, $i = 1,2,\ldots,n$ are iid random variables having Weibull distribution with three parameters. Show that the variable $Y = \min(X_1, X_2, \ldots, X_n)$ also has Weibull distribution.

ii) Show that for t-distribution with k degrees of freedom

$$P[X \le x] = 1 - \frac{1}{2} I_{\frac{k}{k+x^2}}\left(\frac{k}{2}, \frac{1}{2}\right),$$

where

$$I_x(a,b) = \frac{1}{B(a,b)} \int_0^x y^{a-1}(1-y)^{b-1} dy$$

is the incomplete beta function.

iii) Let X_i be independently distributed as $\chi^2_{n_i}$, $i = 1,2$. Obtain the probability density function of $Y = \frac{X_1}{X_2}$.

Question No. 5. **(2X7=14)**

i) Let X_1, X_2 denote a random sample of size 2 from a distribution $N(\mu, \sigma^2)$. Define $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + 2X_2$. Show that the joint *pdf* of Y_1 and Y_2 is bivariate normal distribution with correlation coefficient $\frac{3}{\sqrt{10}}$.

ii) Define $p-$variate normal distribution obtain its probability density function.

iii) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$. Show that $M_{\underline{X}-\underline{\mu}}(\underline{t}) = \exp\left(\frac{1}{2}\underline{t}'\Sigma\underline{t}\right)$, hence find the moment generating function of $\underline{X}$.

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Term End Examinations, July 2023

Programme: M.Sc. (Statistics) **Session: 2022-23**

Semester: II **Max. Time: 3 Hours**

Course Title: Design of Experiments **Max. Marks: 70**

Course Code: SBS ST 01 203 C 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Define the following terms which occur in basic principle in design of experiments replication and local control.

b) Explain the following terms: uniformity trial, size and shape of plots? How does it use in field experiments?

c) What is strip-plot design? Discuss the advantages and disadvantages of strip-plot design.

d) What is a treatment contrast? When are two such contrasts said to be orthogonal?

e) The following are two key-blocks of a lay out plan before randomisation for a 2^4 experiment with factors A, B, C and D

Replication I : (1) abc abd cd

Replication II: (1) abc acd bd.

Identify the confounded effects.

f) Define a Balanced Incomplete Block Design (BIBD). Write the sufficient condition for BIBD.

g) What is meant by confounding in a factorial experiment? Explain the terms complete confounding and partial confounding.

Question No. 2. (2X7=14)

a) Derive the analysis of covariance for a two-way layout with one concomitant variable.

b) What would be efficiency of LSD compared to RBD with columns of LSD as blocks of RBD.

c) Derive the statistical analysis of an RBD with one missing value.

Question No. 3. (2X7=14)

a) Give in detail the analysis of a 2^2 factorial experiment.

b) A 2^5 factorial experiment with factor A, B, C, D and E are arranged in 4 blocks of 8 plots each. If 4 elements of one of the blocks are $(1), ab, cd, e.$ What are the block compositions of the four blocks? Find the factorial effects confounded.

c) Construct a 2^{6-2} fractional factorial design using the design generators I = ACE and I = ACDF. Write down the aliases of main effects.

Question No. 4. (2X7=14)

a) Derived the expression to measure the efficiency of Balanced Incomplete Block Design (BIBD) relative to RBD.

b) Derive the expected values of different sums of squares in the intra-block analysis of variance of a Balanced Incomplete Block Design (BIBD).

c) Give the statistical analysis of CRD and explain its merits and demerits.

Question No. 5. (2X7=14)

a) Give in detail the analysis of a 2^2 factorial experiment.

b) Explain the cross-over design. Write down the analysis of variance table of this design.

c) Give the layouts and statistical analysis of 2 x 2 and 3 x 3 crossover designs. Discuss their advantages and disadvantages.

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Term End Examinations, June 2023

Programme: M.Sc (Statistics)(Re-Appear) **Session: 2022-23**
Semester: III **Max. Time: 3 Hours**
Course Title: Statistical Inference-II **Max. Marks: 70**
Course Code: SBS ST 01 302 C 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Write sort notes on the following terms (i) uniform prior and (ii) noninformative prior.
b) Explain the absolute error loss function? Give a situation where it is applicable.
c) Differentiate between frequentist and Bayesian inference approach.
d) Explain the non-parametric method. How do they differ from parametric methods?
e) Describe the credible and highest posterior density (HPD) interval.
f) Define the following terms with suitable example (i) Kernel (ii) distribution free method.
g) Explain how the sequential test procedure differs from the Neyman-Pearson test procedure.

Question No. 2. (2X7=14)

a) Derive the asymptotically locally invariant prior for exponential family of distributions.
b) Explain approaches for the subjective determination of prior density along with an appropriate example.
c) Obtained Bayes estimators of μ and σ^2 under SELF in the normal probability density function $f(x|\mu.\sigma)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right] \infty<\mu<\infty,\sigma,x>0$ when the joint prior $g(\mu,\sigma)\propto\frac{1}{\sigma^c}, c>0$.

Question No. 3. (2X7=14)

a) Explain the prior and posterior analysis of exponential distribution. Find the Bayes estimator of parameter in exponential distribution. Show that under absolute error loss, the Bayes rule is the median of the posterior distribution.
b) Let $\underline{x}=(x_1,x_2,....x_n)$ be a random sample from the exponential probability density function $f(x|\mu.\theta)=\frac{1}{\theta}\exp\left(-\frac{x-\mu}{\theta}\right)$, $0<\mu<x.\theta>0$, obtained $(1-\alpha)\%$ credible and highest posterior density (HPD) intervals for μ and θ under the vague-prior $g(\mu,\theta)\propto\frac{1}{\theta}$, $\theta>0$.

c) If a sufficient statistics exists for the parameter p of binomial distribution $B(n,p)$ then show the family of conjugate priors exists for p.

Question No. 4. (2X7=14)

a) Explain the Wald-Wolfowitz run test for testing the equality of two distribution functions.

b) Write short notes on the empirical distribution function and K-S test.

c) Describe the median test for the two-sample location problem. Find the distribution of the test statistic and compute its mean and variance under the null hypothesis. How is the test carried out in case of large samples?

Question No. 5. (2X7=14)

a) Define Wald's Sequential Probability Ratio test. Define the OC function and A.S.N function in sequential analysis.

b) Let X have the distribution $f(x,\theta)=\theta^x(1-\theta)^{1-x}$, $x=0,1;\ 0<\theta<1$. For testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$, construct S.P.R.T. and obtain its A.S.N. and O.C. functions.

c) Develop the S.P.R.T. for testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1(>\theta_1)$, based on a random sample of size n from a population with pdf $f(x,\theta)=\frac{1}{\theta}e^{-x/\theta}$, $x>0,\ \theta>0$. Also obtain it's A.S.N. and O.C. functions.

CENTRAL UNIVERSITY OF HARYANA

Term End Examination July 2023

Programme : M.Sc. Statistics		**Session: 2022-23**
Semester : First		*Max. Time:* ***3 Hours***
Course Title : Analysis and Linear Algebra		*Max. Marks : 70*
Course Code : SBS ST 01 101 C 3104		

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Question No. 1. **(4x3.5=14)**

a. Let V be a vector space over F. Prove that the set {v} is L.D iff v = 0.

b. The union of two closed sets is a closed set.

c. Show that the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+.........$is convergent.

d. If v is a linear combination of $v_1, v_2, ...v_n$ then show that $v_1, v_2, ...v_n$, v are L.D vectors.

e. Show that the sequence, $\{S_n\}$, where $S_n = \frac{1}{1!}+\frac{1}{2!}+.........+\frac{1}{n!}$, $\forall\, n \in N$ is convergent.

f. Show that the analytic function f(z) is constant if

(i) Real part f(z) is constant.

(ii) Imaginary part of f(z) is constant.

g. Write down the quadratic form corresponding to matrix A $\begin{bmatrix} 2 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$

Question No. 2 **(2x7=14)**

a. A positive term series $\sum \frac{1}{n^p}$ is convergent if and only if p >1.

b. If $\sum u_n$ is a positive term series, such that $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = l$, then the series

(i) Converges, if $l < 1$,

(ii) Diverges, if $l > 1$

(iii) The test fails to give any definite information, if $l = 1$.

c. Show that $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+.......+\frac{1}{\sqrt{n^2+n}}\right]=1$

Question No. 3 **(2x7=14)**

a. Show that the function $f(z) = u + iv$ where

$$f(z)=\begin{cases}\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & z\neq 0\\ 0 & z=0\end{cases}.$$ Satisfies the Cauchy Riemann equations at z=0.

Is the function analytic at z=0. Justify your answer.

b. Evaluate $\int_C \frac{e^z}{(z-1)(z-4)}dz$ where c is the circle $|z|=2$ by using cauchy's integral formula.

c. Evaluate by contour integration $\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta - n\theta)d\theta$.

Question No. 4 **(2x7=14)**

a. Reduce to row echelon form the matrix A= $\begin{bmatrix}1 & -2 & -1 & 4\\ 2 & -4 & 3 & 5\\ -1 & 2 & 6 & 7\end{bmatrix}$. Also find the row rank of A.

b. Consider the following block diagonal matrix A= $\begin{bmatrix}8 & -7 & 0 & 0 & 0\\ 1 & -1 & 0 & 0 & 0\\ 0 & 0 & 3 & 1 & 0\\ 0 & 0 & 5 & 2 & 0\\ 0 & 0 & 0 & 0 & 4\end{bmatrix}$ find A^{-1}.

c. Verify Cayley Hamilton theorem for the matrix A= $\begin{bmatrix}0 & 0 & 1\\ 1 & 2 & 0\\ 2 & -1 & 0\end{bmatrix}$. Hence find A^{-1}.

Question No. 5 **(2x7=14)**

a. Show that $\{e_1, e_2, e_3\}$ where $e_1 = \{1, 0, 0\}$ $e_2 = \{0, 1, 0)$ and $e_3 = \{0, 0, 1\}$ is the orthonormal subset of R^3.

b. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2^2 + 4x_2x_3 - x_3^2$ to diagonal form.

c. Which of the following subsets of the vector space of all polynomials are L.I or L.D

(i) $S= \{x^2-1, x+1, x-1\}$

(ii) $S= \{1+x, x+x^2, x^2+x^3, x^3+x^4, x^4-1\}$

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations July 2023

Programme : M.Sc. Statistics
Semester : Third
Course Title : Stochastic Processes
Course Code : SBS ST 01 301 DCE 3104

Session: 2022-23
Max. Time: 3 Hours
Max. Marks: 70

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and half Marks.
2. Question no. 2 to 5 have three parts and student need to answer any two parts of each question. Each part carries seven marks.

Question No. 1.

(4x3.5=14)

a. Define periodic state, ergodic state and ergodic chain.

b. Obtain the probability generating function for the poisson distribution.

c. Define random walk model with the help of suitable example.

d. Consider the three state Markov chain having transition probability matrix

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

. Classify the Markov chain.

e. Show that for a Galton Watson branching process, for r, n = 0,1, 2, ….

$$E\{X_{n+r}|X_n\} = X_n m^r$$

f. Define Absorption probabilities. The state transition matrix of the Markov chain is given by the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let's define a_i as the absorption probability in state 3 if we start from state i. Obtain a_i for i= 0, 1, 2, 3.

g. Show that sum of two independent Poisson processes is a Poisson process.

Note: Question number **Two to Five** have three sub parts and students need to answer **any two sub part** of each question. Each sub part carries **seven marks.**

Question No. 2 **(2x7=14)**

a. Prove that in an irreducible chain all the states are of same type.

b. Define Stochastic Process. Discuss the types of stochastic process with the help of suitable examples.

c. Consider the Markov chain given below and find the two step transition matrix. Also find P_{01}^{2} and $\Pr\{X_2 = 1, X_0 = 0\}$

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Question No. 3 **(2x7=14)**

a. Define Branching process and also obtain mean and variance of the size of the n^{th} generation.

b. In a classical gambling problem find the expected duration of the game.

c. Prove that if m=1, the probability of ultimate extinction is 1. If m > 1 the probability of ultimate extinction is the positive root less that unity of the equation p(x) = x.

Question No. 4 **(2x7=14)**

a. Obtain the probability generating function of the birth and death process.

b. Define simple birth process and also obtain its probability generating function.

c. Discuss in detail the generalization of the Poisson process.

Question No. 5 **(2x7=14)**

a. Define Semi Markov process also prove that with probability 1

$\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$, where $\mu = E(X_n) \leq \infty$

b. State and Prove the Central Limit Theorem for Renewals.

c. If we have

$$Pr(Y(t) \leq x) = F(t-x) - \int_0^t [1 - F(t+x-y)]dM(y)$$

If in addition, F is non-lattice, then prove that

$$\lim_{t\to\infty} \Pr\{Y(t) \leq x\} = \frac{1}{\mu}\int_0^x [1 - F(y)]dy.$$

CENTRAL UNIVERSITY OF HARYANA

Term End Examination June/July 2023

Programme : M.Sc. Statistics	**Session: 2022-23**
Semester : First	**Max. Time: 3 Hours**
Course Title : Sampling Techniques	**Max. Marks: 70**
Course Code : SBS ST 01 104 C 3104	
Regular/Reappear: Reappear	

Instructions:

1. Question no. 1 has seven parts and students need to answer any four. Each part carries three and a half Marks.
2. Question number two to nine carries 14 marks each and student need to attempt any four questions from eight questions.

Question No. 1. **(7x2=14)**

a. What are the principal steps in a sample survey?

b. Under what circumstances census surveys are preferred over sample surveys?

c. Define non-probability sampling. Also, give its disadvantages.

d. Explain simple random sampling with its properties.

e. Discuss the advantages and disadvantages of systematic sampling.

f. What is the regression method of estimation?

g. Describe cluster sampling highlighting its advantages.

Note: Question number **Two to Five** have three subparts and students need to answer **any two sub part** of each question. Each subpart carries **seven marks.**

Question No. 2 **(2x7=14)**

a. Prove that in simple random sampling sample mean $\bar{y}$ is an unbiased estimator of the population mean $\bar{Y}$ with its sampling variance $Var(\bar{y}) = \frac{(N-n)}{N}\frac{S^2}{n}$.

b. Prove that in simple random sampling, $s^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2 \Big/ (n-1)$ is an unbiased estimator of $S^2 = N\sigma^2/(N-1)$.

c. What do you understand by the terms "sampling frame" and "finite population correction factor"? Explain the basic principles of sample survey.

Question No. 3 **(2x7=14)**

a. For stratified random sampling without replacement, prove that, the sample estimator $\bar{y}_{st} = \sum_{h=1}^{L} N_h \bar{y}_h / N$ is an unbiased estimator and its sampling variance is given by $Var(\bar{y}_{st}) = \sum_{h=1}^{L} (1-f_h) W_h^2 S_h^2 / n_h$.

b. Show that with stratified random sampling without replacement, an unbiased estimator of the variance of $\bar{y}_{st} = \sum_{i=1}^{k} N_i \bar{y}_i / N$ is given by $\hat{Var}(\bar{y}_{st}) = \sum_{i=1}^{k} N_i (N_i - n_i) s_i^2 / N^2 n_i$.

c. Describe the method of systematic sampling and prove that systematic sampling is more efficient than SRSWOR if $\rho \leq \frac{-1}{(nk-1)}$.

Question No. 4 **(2x7=14)**

a. Show that the first approximation to the relative bias of ratio estimator in simple random sampling without replacement is given by

$$B(\hat{R}) \square \frac{(1-f)}{n} R(C_x^2 - \rho C_x C_y).$$

b. Show that the first approximation to the relative bias of ratio estimator in simple random sampling without replacement is given by

$$\frac{B(\hat{R})}{R} \square \frac{1-f}{n\bar{X}\bar{Y}} (RS_x^2 - \rho S_x S_y) = \frac{1-f}{n}(C_x^2 - \rho C_x C_y).$$

c. Discuss the comparison of regression estimator with ratio estimator and mean per unit.

Question No. 5 **(2x7=14)**

a. Prove that in simple random sampling without replacement of n clusters each containing M elements from a population of N clusters, the sample mean $\bar{y}_n$ is an unbiased estimator of the population mean $\bar{Y}$ and its variance is given by

$$Var(\bar{y}_n) \square (1-f)\frac{S_M^2}{n}[1 + (M-1)\rho].$$

b. What do you mean by probability proportional to size sampling with replacement? Give a method of selecting such a sample, suggest an unbiased estimate of population mean and also find its variance.

c. Describe a two stage sampling and give its advantages. For a two stage sampling with equal first stage units, obtain an unbiased estimate of the population mean and also derive its variance. Also, find the estimate of the variance.

CENTRAL UNIVERSITY OF HARYANA

Second Semester Term End Examinations July 2023

Programme: PG

Semester: Second | **Session: 2022-23**

Course Title: Biostatistics | **Max. Time: 3 Hours**

Course Code: SBS ST 01 202 GE 3104 | **Max. Marks.: 70**

Instructions

- Question no. 1 has seven parts and students need to answer any four. Each part carries three and a half Marks.
- Questions no. 2 to 5 have three parts and students need to answer any two parts of each question. Each part carries seven marks.

Q. 1. $(4 \times 3.5 = 14)$

a. What do you mean by Primary data & Secondary Data. Discuss the sources of secondary data.

b. Discuss Attribute & Variable with example. Write the scales of Measurement.

c. What do you mean by Table in Statistics? Write short notes on parts of table with layout.

d. What is Ogive? Which measure is determined from Ogive? How can you locate Positional value from Ogive?

e. Note down the characteristics of Good Average. Show that mean & median of first 20 natural numbers are 10.5.

f. Discuss Probability & Non-Probability Sampling.

g. Discuss CRD & RBD & distinguish between them.

Q. 2. $(2 \times 7 = 14)$

a. Discuss different ways of collecting Primary data with merits & limitations.

b. What is Classification of data? Discuss different types of classification with example. What is the difference between classification & tabulation of data?

c. What are the scales of measurement? Discuss them with at least two examples.

Q. 3. (2 × 7 = 14)

a. What are different measures of central tendency. Arithmetic Mean is an Ideal Average. Comment.

b. Find the missing frequency in the following distribution whose median mark is 38.5 & mean mark is 40.6.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	8	?	25	40	?	22

c. Write Short notes on Stem & leaf Chart. What is Histogram? How can you construct a Histogram for unequal classes? Discuss with example.

Q. 4. (2 × 7 = 14)

a. Describe the steps involved in sample survey.

b. Discuss merits & demerits of Sample Survey & Complete enumeration. When do we prefer Census survey?

c. What are Sampling distributions? Discuss the different tests of large-scale survey.

Q. 5. (2 × 7 = 14)

a. What are the Principles of Design of Experiment. Discuss them in detail.

b. Write Short notes on: a) Replication, b) ANOVA, c) Fertility Counter Map

c. Discuss in Detail the LSD with layout & ANOVA.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, July 2023

Programme : M. Sc. Statistics
Semester : II
Course Title : Statistical Inference-I
Course Code : SBS ST 01 201 C 3104

Session: 2021-2022
Max. Time : 3 Hours
Max. Marks : 70

Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. **(4X3.5=14)**

a) Define the Likelihood Ratio Test with example.

b) State and prove Lehmann-Scheff e' Theorem.

c) Discuss the terms "Point estimation", "Interval estimation" and testing of hypothesis with at least one example each.

d) Explain the methods of maximum likelihood and write down its small sample properties.

e) Find the *mle* of θ if sampling from Laplace distribution.

f) To test $H_0: \mu = 200$, $H_A: \mu > 200$. A random sample of size 20 is drawn from a population following normal distribution with unknown mean μ and variance 80. H_0 is accepted if $\bar{x} \leq 204$.

g) Explain the following with example

i) Statistical hypothesis
ii) Null hypothesis
iii) Critical region
iv) Type I error and Type II error

Question No. 2. **(2X7=14)**

a) If T_1 and T_2 are two minimum variance unbiased estimators for a parameter θ, then prove that $T_1 = T_2$.

b) Suppose X and Y are independent random variables with the same unknown means μ. Both X and Y have variance as 36. Let $T = aX + bY$ be an estimator of μ.

i) Show that T is an unbiased estimator of μ if $a + b = 1$.

ii) If $a = 1/3$ and $b = 2/3$, what is the variance of T.

iii) What choice of a and b minimizes the variance of T subject to the requirement that T is an unbiased estimate of μ.

c) Let $x_1, x_2, \cdots, x_n$ be *iid* random variable with mean θ and finite variance σ^2, show that

$$T = \frac{2}{n(n+1)} \sum_{i=1}^{n} i x_i$$ is a consistent estimator of θ.

Question No. 3. **(2X7=14)**

a) Theoretical probability in the four cell of a multinomial distribution are $\frac{2+\theta}{4}$, $\frac{1-\theta}{4}$, $\frac{1-\theta}{4}$ and $\frac{\theta}{4}$, where as the observed frequencies are 108, 27, 30 and 8 respectively. Estimate θ by max. likelihood method and give the standard error of the estimate.

b) Show that Cauchy with parameters 0 and θ has no monotonic likelihood ratio family of distribution.

c) For double Poisson distribution with probability mass function $P[x=x]=\frac{1}{2}\frac{e^{-\lambda_1}\lambda_1^x}{x!}+\frac{1}{2}\frac{e^{-\lambda_2}\lambda_2^x}{x!}$, $x=0,1,2,\cdots$. Find the estimates for λ_1 and λ_2 by the method of moments.

Question No. 4. **(2X7=14)**

a) A random sample of size n is given from a $B(m,p)$ population to test $H_0: p=1/2$ vs $H_A: p=1/4$ (m is fixed and known). Find MP size α test.

b) Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_n$ be independent random sample from $N(\theta_1, \sigma_1^2)$ and $N(\theta_2, \sigma_2^2)$ respectively. Obtain LRT of $H_0: \theta_1=\theta_2, \sigma_1^2>0, \sigma_2^2>0$ against $H_1: \theta_1 \neq \theta_2, \sigma_1^2>0, \sigma_2^2>0$ when σ_1^2 and σ_2^2 are unknown.

c) Let $X_1, X_2, \cdots, X_n$ be iid $U(\theta-1/2, \theta+1/2)$, $\theta \in \Re^1$. Show that the statistic $T(X)=(X_{(1)}, X_{(n)})$ is minimal sufficient but not complete.

Question No. 5. **(2X7=14)**

a) Given a random sample from the distribution with pdf $f(x;\theta)=\theta e^{-\theta x}$, $x>0$, show that there exist no UMP test for testing $H_0: \theta=\theta_0$ vs $H_A: \theta \neq \theta_0$.

b) For testing $H_0: \theta=\theta_0$ vs $H_A: \theta=\theta_1$, [i.e. i) $\theta_1>\theta_0$ ii) $\theta_1<\theta_0$] in the case of a normal population $N(\theta, \sigma^2)$, σ^2 is known. Obtain MP size α test. Can you say that it is also UMP test? Also find power of the test.

c) Let $X_1, X_2, \cdots, X_n$ be a random sample from the $N(\theta, \sigma^2)$ population. Find $(1-\alpha)\%$ level of confidence interval.